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Co-Neutrosophic Multi-Domination in Co-Neutrosophic Graphs

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Abstract

The goal of this article is to examine the ideas of co-neutrosophic multi-dominating (CNMD) set, number γ_{CNMD} in a co-neutrosophic graph (G_{CN}) and their inverses CNMD^{-1},γ_{CNMD}^{-1} respectively. For specific classes of co-neutrosophic graphs (G_{CN}), the γ_{CNMD} and γ_{CNMD}^{-1} calculated and constraints on the corresponding γ_{CNMD} drived. Also, some example for evaluation the inverse co-neutrosophic domination provided.

Keywords: Single value neutrosophic graph, co-neutrosophic graph, domination set, domination number.

الهيمنة المتعددة الضد نيوتروسوفيك في الرسوم البيانية الضد نيوتروسوفيك

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الخلاصة

الهدف من هذه المقالة هو ادخال وفحص فكرة مجموعات متعددة الهيمنة (CNMD) وحساب اعداد الهيمنة في رسم بياني ضد النيوتروسوفيك (G_{CN}) وتطبيق الفكرة على فئات محددة من الرسوم البيانية الضد نيوتروسوفيك، وحساب اعداد الهيمنة لـ (CNMD) ونشتق القيود على اعداد الهيمنة المقابلة لها أيضًا، قدمنا ايضا مفهوم الهيمنة العكسية للرسوم البيانية ضد النتروسوفيك.

الكلمات المفتاحية: رسم بياني نيوتروسوفيك ذو قيمة واحدة، رسم بياني نيوتروسوفيك مشترك، مجموعة السيطرة، رقم السيطرة.

1.Introduction

The challenge of identifying the bare minimum of queens required to cover a nxn chessboard gave rise to the mathematical study of dominant sets in graphs in the 1850s.Different writers have explored more than 50 different kinds of domination factors [1]. In 1965[2], Zadeh developed the concept of a fuzzy set as a framework for mathematically capturing ambiguity and imprecise information. Fuzzy analogs of key graph theoretic notions, including path, cycle, and connectedness, were proposed by Rosenfeld[3].A. Somasundaram and S. Somasundaram researched the idea of domination in fuzzy graphs, and A. Somasundaram presents the ideas of independent domination, total domination, and linked domination of fuzzy graphs [4].K. T. Atanassov[5] introduced the concept of Intuitionistic Fuzzy (IF) relations and Intuitionistic Fuzzy Graphs(IFGs).Muhammad A. [6] introduced ideas like linked anti-fuzzy graphs, constant anti-fuzzy graphs, and others after learning about anti-fuzzy structural graphs.Muthuraj R. and Sasireka A. construct the concepts of dominance on anti-fuzzy graph and linked dominance on anti-fuzz in their study on the topic. Dr.C. Rajan & A. Senthil Kumar define the dominating set and dominating number of the single valued neutrosophic graph G= (A, B), as well as some limits on its dominating number [7]. Ghobadi, Soner, and Mahyoub introduced the inverse dominating set in fuzzy graphs [8]. In this paper we introduce the conception of co neutrosophic multi dominating set and number in co neutrosophic graphs.

2.Preliminaries

Definition 2.1.[9,11] A graph $G_{CN} = (A, B)$ on an underline simple graph $G^*_{CN} = (V, E)$ *is called a* single-valued co-neutrosophic graph (SVCNG),

where $A : V \to [0,1]$ be SVCN vertex set of G
and $B : V \times V \to [0,1]$, SVCN edge set of G
satisfy the following

$$T_B(xy) \geq \max\{T_A(x), T_A(y)\}$$
$$I_B(xy) \geq \max\{I_A(x), I_A(y)\}$$
$$F_B(xy) \leq \min\{F_A(x), F_A(y)\} \text{for all } x, y \in V .$$

The graph .1 below is an example of SVCNG.

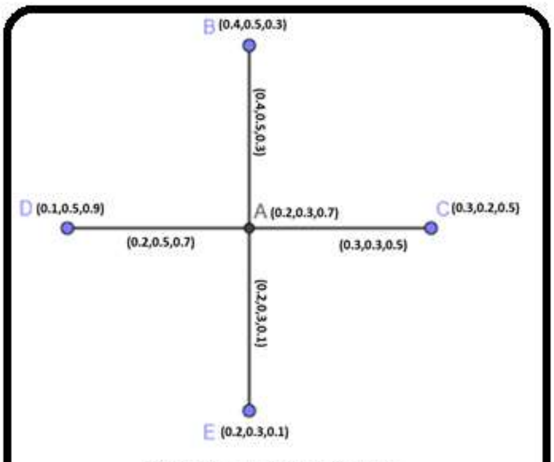

Figure 1: co-neutrosophic graph

Definition 2.2.[12]. A path P_n^{CN} $in\ G_{CN}$ is a string of various vertices $(x_1, x_2, x_3, \dots, x_n)$ such that
$T_B(x_i, x_{i+1}) > 0$ for $1 \le i \le n$

Definition2.3. A connected co-neutrosophic graph G_{CN} is one in which any two vertices in it have a neutrosophic path between them.

Definition 2.4. A complement of (SVCN) graph G_{CN} = (A, B) on G* is a (SVCN) graph denoted as $\overline{G_{CN}} = (\bar{A}, \bar{B})$ such that $\bar{A}$ = A i.e., $T_A(x) = \bar{T}_A(x), I_A(x) = \bar{I}_A(x)$,
$F_A(x) = \bar{F}_A(x)$and
$\bar{T}_{\bar{B}}(x,y) = \max\big(T_A(x), T_A(y)\big) - T_B(x,y)$
$\bar{I}_{\bar{B}}(x,y) = \max(I_A(x), I_A(y)) - I_B(x,y)$
$\bar{F}_{\bar{B}}(x,y) = \min(F_A(x), F_A(y)) - F_B(x,y)$ for all $xy \in E$.
Definition 2.5. [10] Consider a neutrosophic graph $G_N = (A, B)$ on underline simple graph $G_N^* = (V, E)$
1) The relation between any pairs of elements (vertices) of V is called edge and denoted by $e \in E$.
2) an edge $e = xy\ in\ G_N$ is said to effective edge if
$$T_B(xy) = \max\big(T_A(x), T_A(y)\big), I_B(xy) = \max\big(I_A(x), I_A(y)\big)\ and$$
$$F_B(xy) = min\big(F_A(x), F_A(y)\big)$$
3) A (SVCN) graph $G = (A, B)$ is said to strong graph if for every edge $e = (xy) \in E$ is an effective edge.

Definition 2.6.[10] A (SVCN) graph $G = (A, B)$ is said to be complete graph if for every $x, y \in V\ \exists$ an effective edge $e = (xy) \in E$.
Definition 2.7. Let $G^* = (V, E)$ be underlining graph of a (SVCN) G. then $w \in V$ is said to be effectively isolated vertex if $xw \in E$ is not effective $\forall\ x \in N(w)$,

In private case if $T_B(x,w) = I_B(x,w) = F_B(x,w)$= 0 ∀ x ∈ $V - \{w\}$ then w is called strongly isolated (isolated in the underling graph),

Definition 2.8.[8] independent Co-neutrosophic set S is a subset of $V\ (G_{CN}\ )$ where $T_B(x,w) \neq \max(T_A(x), T_A(w))$,$I_B(x,w) \neq \max(I_A(x), I_A(w))$ and $F_B(x,w) \neq \min(F_A(x), F_A(W))\ \forall\ x, w \in S$.
In addition, it said to be maximal if there exist no independent Co-neutrosophic set $Z \subset V$ *and* $|S| < |Z|$.The independence number
$\boldsymbol{\beta_o}$ (G_{CN}) = max $\{S_i: S_i\ maximal$ independent Coneutrosophic set}.

Definition 2.9. [8]A Co-neutrosophic graph G_{CN} = (A, B) is called a **unimodal** if $(T_A(x), I_A(x), F_A(x))$ = (k, k, k) =$(T_B(x,y), I_B(x,y), F_B(x,y))$while if $(T_A(x), I_A(x), F_A(x))$ = (c, c, c) ∀ x ∈ $V\ (G_{CN}\ )$ then G_{CN} is known as **x- nodal**, k, c ∈ [0.1]

Definition 2.10.[3] let G_{CN} a co-neutrosophic graph then:

1) A **co-neutrosophic vertex cover set A** of G_{CN} is a subset of V such the for each effective edge $e = uv\ \in E, at\ least\ u \in A\ or\ v \in A$
2) A co-neutrosophic vertex cover (α_O) number =max $\{A_i$: A_i is Co-neutrosophic vertex cover set of G_{CN} with minimum numbers of vertices}
3) $u \in V(G_{CN})$ is known as end point if there exist only one vertex $v \in V$ such that uv is effective edge.

Definition 2.11.[5] $\mathcal{D} \subseteq V\ (G_{CN})$ is called **co-neutrosophic dominating (CND) set** of G_{CN}.
If $\forall u_i \in V - \mathcal{D}\ \exists\ v_j\ \in \mathcal{D}$ such that

$$T_B(u_i v_j\ ) = \max\{T_A(u_i), T_A(v_j\ )\}$$
$$I_B(u_i v_j) = \max\{I_A(u_i), I_A(v_j\ )\}$$

$F_B(u_i v_j) = \min\{F_A(u_i), F_A(v_j\ )\}$for all $v_i\ , v_j\ \in\ V$.

Definition 2.12.[5] The minimum co-neutrosophic dominant (MCND) set is the (CND) set D of G_{CN} with the fewest number of vertices. If there exist no $\mathcal{D}' \subseteq \mathcal{D}$ that is CND set, then D is known as the minimum CND set of G_{CN} and take the maximum cardinality for all MCND sets is known as a co-neutrosophic Domination (CND) number of G_{CN} and denoted by $\gamma_{\text{CND}}\ (G_{CN})$ or simply γ_{CND}.

3. Co-neutrosophic *multi-domination number* G_{CN}.

In this section, we introduce the co-neutrosophic multi-***domination*** CNMD set and number of G_{CN}, and Co-neutrosophic vertex covering with suitable illustrations, and we go over certain CNMD number of G_{CN} by the effective edge attributes.

Definition 3.1. Let $G_{CN} = (A, B)$ be a co-neutrosophic graph on the underline simple graph $G^*_{CN} = (V, E)$, then a non-empty set $\mathcal{D} \subseteq V$ is called (CNMD) set in G_{CN} if ∀ $x \in V - \mathcal{D}$
There is more than one neighbors in $\mathcal{D}$ *i.e.*, ∃ at least two vertices y, z ∈ D such that both of xy and xz are effective edges in E *i.e.*$T_B(x,y) = T_A(x) \wedge T_A(y)$,

$F_B(x,y) = F_A(x) \wedge F_A(y)$, and $I_B(x,y) = I_A(x) \wedge I_A(y)$similarly for *xz*.
Definition 3.2. A minimal *CNMD-set D of G_{CN} is a CNMD* set that has the fewest number of vertices possible.
Definition 3.3. *A* CNM*D set $\mathcal{D}$ of G_{CN}* is called minimum co-neutrosophic multi-domination *MCNMD set in G_{CN} if $\nexists$ $D' \subset D$ such that, D' as* CNM*D set of G_{CN}* .

Definition 3.4.

1) The cardinality (Score) of any co-neutrosophic
$$A(x) = \{< x, T_A(x), I_A(x), F_A(x) >\} is\ |A(x)| = \frac{1 + T_A(x) + I_A(x) - F_A(x)}{3}$$
2) The cardinality of order of the co-neutrosophic graph $|O_{CN}| = \sum_{v_i \in V} |v_i|$
3) The maximal neutrosophic cardinality over all MCNMD sets is known as a CNMD number of G_{CN} and is represented by the symbol $\gamma_{\text{CNMD}(}\ G_{CN})$

Example3.1. Consider a co-neutrosophic graph G_{CN} = (A, B), which given in figure (2), where we have D_1 = {A, C, E}, D_2 = {B, D, F} are MCNM*D* sets also are minimal CNM*D* sets of G_{CN}.
Hence, *by* $\gamma_{\text{CNMD}(}\ G_{CN})$ = *max* [| D_1 |, | D_2 |] = *max* [|(1,1.6,1.8)|,|(1,1.6,1.4)|] = *max* (1.8,2.2) =2.2

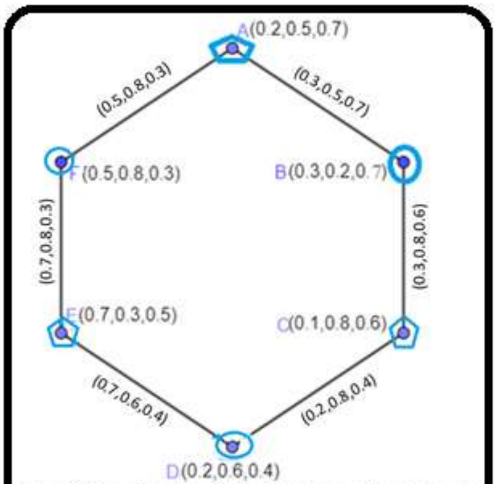

Figure 2: Co-neutrosophic multi-domination in co-neutrosophic graphs

Preposition 3.1. *Let* $G_{CN} \cong k_n^{CN}$, n≥ 3 be a complete co-neutrosophic graph.
then $\gamma_{\text{ CNMD}}(k_n^{CN})$ = max $\{|T_A(u_i), I_A(u_i), F_A(u_i)|, |T_A(u_j), I_A(u_j), F_A(u_j)|\}$ $\forall u_i, u_j \in V(k_n^{CN})$**. *i, j=1,…,*** *n such that* $u_i \neq u_j$
Proof: Given $G_{CN} \equiv k_n^{CN}$ be a complete co-neutrosophic graph, then
$$T_B(u_i u_j) = \max\{T_A(u_i), T_A(u_j)\}$$
$$I_B(u_i u_j) = \max\{I_A(u_i), I_A(u_j)\}$$
$F_B(u_i u_j) = \min\{F_A(u_i), F_A(u_j)\}$for all $u_i, u_j \in V(k_n^{CN})$

As a result, every vertex in k_n^{CN} has dominance over every other vertex in k_n^{CN}.Any set in k_n^{CN}that has two vertices, such as $\{u_1u_2\}$, will therefore be of the form CNMD set of k_n^{CN}Hence $\gamma_{\text{ CNMD}}(k_n^{CN})$ = max $\{|T_A(u_i), I_A\ (u_i), F_A\ (u_i)|, |T_A(u_j), I_A\ (u_j), F_A\ (u_j)|\}$

Preposition 3.2. *Let G_{CN} be a co-neutrosophic star graph then*
$\gamma_{\text{ CNMD}}(k_{n,M}^{CN})$ = $|O_{CN}|$ $-|T_A(u), I_A(u), F_A(u)\ |$, u is root of the star.
Proof: Given S_n^{CN} be a strong co-neutrosophic star graph with v as a root of S_n^{CN} then for every vertex in star S_n^{CN} except the vertex $\{v\}$ has a single neighbor. Then $V-\{v\}$ is only CNMD set of S_n^{CN} *, therefore* $\gamma_{\text{ CNMD}}$ (GCN) = $|V-\{v\}|$=$|O_{CN}|$ $-|T_A(u), I_A(u), F_A(u)\ |$, v being a root vertex.
Proposition 3.3: If C_n^{CN} be strong co-neutrosophic cycle graph with n vertices $\{u_1, u_2,.., u_n\}$ *then*

$$\gamma_{\text{ CNMD}}\ (C_n^{CN}) = \begin{Bmatrix} max\left\{\sum_{i=0}^{\frac{n}{2}-1}|T_A(v_{j+2i}), I_A\ (v_{j+2i}), F_A\ (v_{j+2i})|; j=1,2\right\} & \text{if n is even} \\ max\left\{\sum_{i=0}^{\left\lfloor\frac{n}{2}\right\rfloor}|T_A(v_{j+2i}), I_A\ (v_{j+2i}), F_A\ (v_{j+2i})|; j=1,2,\ldots n\right\} & \text{if n is odd}\end{Bmatrix},$$
$$j+2i \ mod\ n$$

Proof: Let C_n^{CN} *with* $\{u_1, u_2,.., u_n\}$ be a strong cycle, so there exist two cases:
Case1: As a result, $\forall\ u \notin \mathcal{D}$, u have to have couple of neighbors in $\mathcal{D}$ and this satisfy if there are two edges between any pair of vertices in D.
Thus, $\mathcal{D}_j = \{u_j\ , where\ all\ j\ is\ taken\ ethier\ even\ or\ odd\}$ which means that there are only two different CNMD which are $\mathcal{D}1$ and $\mathcal{D}2$ of odd vertices and even vertices respectively.
Therefore,

$\gamma_{\text{ CNMD}}\ (C_n^{CN}) = max\left\{\sum_{i=0}^{\frac{n}{2}-1}\ |A(v_{j+2i})|\ ; j+2i\ mod\ n\ , j=1,2\right\}$, where $A(v_{j+2i}) = \{T_A(v_{j+2i}), I_A(v_{j+2i}), F_A(v_{j+2i})\}$
Case 2: if n is odd.
Each vertex in V-D has pair of neighbors, similar to case 1, so for each j the minimum one of the last two vertices in the cycle also must be in $\mathcal{D}_j$,so there are n distinct CNMD sets rely on j; j = 1,2, …, n. It is simple to conclude that all of Dj's sets are CNMD sets. Therefore,

$$\gamma_{\text{ CNMD}}\ (C_n^{CN}) = max\left\{\sum_{i=0}^{\left\lfloor\frac{n}{2}\right\rfloor}|T_A(v_{j+2i}), I_A\ (v_{j+2i}), F_A\ (v_{j+2i})|; j=1,2,\ldots n\right\}$$

The proof comes from the two cases mentioned above. □
Proposition3.4. *Let* GCN $\equiv P_n^{CN}$be a strong co-neutrosophic path n vertex (v_1, v_1, …, v_n) then

$$\gamma_{\text{ CNMD}}(P_n^{N}) = \begin{Bmatrix} \sum_{i=0}^{\left\lfloor\frac{n}{2}\right\rfloor}|T_A(v_{2i+1}), I_A\ (v_{2i+1}), F_A\ (v_{2i+1})| & if\ n\ is\ odd \\ max\left\{\begin{matrix}|v_{2i-1}| + \sum_{j=0}^{\frac{n}{2}-1}\left||T_A(v_{2i+2j}), I_A\ (v_{2i+2j}), F_A\ (v_{2i+2j})|\right|, \\ i=1,2,\ldots,\frac{n}{2}; and\ 2i+2j \equiv t(mod(n+1)),\end{matrix}\right\} & if\ n\ is\ even\end{Bmatrix}$$

.

Proof. For any path the CNMD must contain both end vertices of it, then there are pair of distinct cases depend on n as follows.

Case 1. When n is odd, $\mathcal{D}$ contain a sequence of alternate vertices starting from the first vertex and ending with the last vertex of the path i.e., $\mathcal{D}$ = $\{v_{2i+1}$, i = 0, …. .$\left\lfloor\frac{n}{2}\right\rfloor$, it is obvious that $\mathcal{D}$ is CNMD set. Also, $\mathcal{D}$ is MCNMD set, since if there exist a set F with a smaller number of vertices than set D, then F is not CNMD set. Thus, $\gamma_{\text{ CNMD}}(P_n^N) = \sum_{i=0}^{\left\lfloor\frac{n}{2}\right\rfloor}|v_{2i+1}|$

Case 2. If n is even, the alternating sequence technique is not enough because it ignores one of the two ends, so another vertex must be added to include both end vertices, thus the vertices v_1 and v_1 must belongs to every MCNMD set. In this case, every MCNMD set must contain two neighboring vertices, and each subsequent pair of two vertices in D must have a distance of two edges. Now, for each adjacent pair v_i and v_{i+1} of vertices belongs to $\mathcal{D}$ the other vertices of the path which belongs to $\mathcal{D}$ must be alternate in both sides before vi and after v_{i+1},so let

$$D_i = \{\{v_{2i-1}, v_{2i+2j}, j = 0, \ldots, \tfrac{n}{2} - 1\}, i = 1, \ldots \tfrac{n}{2}\ and\ 2i + 2j \equiv t\ (mod\ (n + 1))\}.$$

It is explicit that each of set D_i is MCNMD set. Thus,

$$\gamma_{\text{ CNMD}}\ (P_n^N)=max\left\{|v_{2i-1}| + \sum_{j=0}^{\frac{n}{2}-1} \left|\left|T_A(v_{2i+2j}), I_A\ (2j), F_A\ (v_{2i+2j})\right|\right|, i = 1,2, \ldots, \tfrac{n}{2}; and\ 2i + 2j \equiv t(mod(n + 1)),\right\}$$

Example 3.2. Suppose that P_6^{CN} , P_7^{CN} are given in a figure (3) below as $G_a\ and\ G_b$ respectively

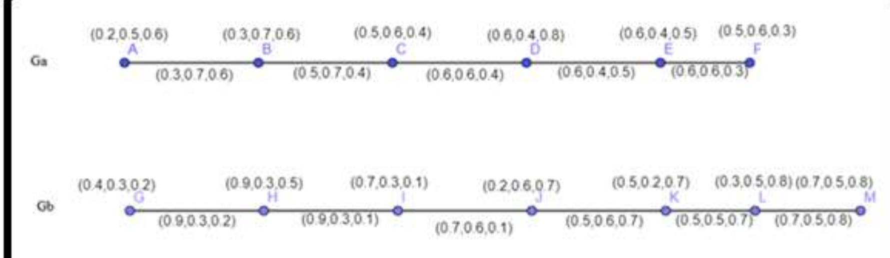

Figure 3

The MCNMD sets of P_6^{CN} are D_1 = {A, B, D, F}, D_2 = {A, C, D, F}, D_3= {A, C, E, F}

$|A| = \left|\frac{1+0.2+0.5-0.6}{3}\right| = 0.36667$,$|B| = \left|\frac{1+0.3+0.7-0.6}{3}\right| = 0.46667$,$|C| = \left|\frac{1+0.5+0.6-0.4}{3}\right| = 0.56667$

$|D| = \left|\frac{1+0.6+0.4-0.8}{3}\right| = 0.4$,$|E| = \left|\frac{1+0.6+0.4-0.5}{3}\right| = 0.5$,$|F| = \left|\frac{1+0.5+0.6-0.3}{3}\right| = 0.6$

$|G| = \left|\frac{1+0.4+0.3-0.2}{3}\right| = 0.5$, $|H| = \left|\frac{1+0.9+0.3-0.5}{3}\right| = 0.56667$, , $|I| = \left|\frac{1+0.7+0.3-0.1}{3}\right| = 0.6333$

, $|J| = \left|\frac{1+0.2+0.6-0.7}{3}\right| = 0.36667$, , $|K| = \left|\frac{1+0.5+0.2-0.7}{3}\right| = 0.3333$, , $|L| = \left|\frac{1+0.3+0.5-0.8}{3}\right| = 0.3333$,, $|M| = \left|\frac{1+0.7+0.5-0.8}{3}\right| = 0.46667$

The MCNMD sets of P_6^{CN} (N=6 Even number) are D_1 = {A, B, D, F}, D_2 = {A, C, D, F}, $\mathcal{D}3$ = {A, C, E, F}

$|D_1|$=1.4333, $|D_2|$=*1. 9333*, $|D_3|$=*2.0333*
γ_{CNMD} (GCN) = max [$|D_1|$, $|D_2|$, $|D_3|$= max [1.4333, *1.9333*, *2.0333*] = 2.0333
While MCNMD sets of P_7^{CN} (N=7 Odd number) is just $\mathcal{D}1$ = {G, I, K, M},
$|\mathcal{D}1|$=1.93327

Proposition 3.5. Every CNMD *set of* G_{CN} = (A, B) *is* CND *set of* G_{CN} .
Proof: The proof is come directly from the definition of MCNMD set.

Proposition 3.6. For any strong co-neutrosophic tree graph T_n^{CN}.if S be a set of all leaf vertices v_i then

$\sum_{v_i \in S}|v_i| \leq \gamma_{CNMD} < O_N$.

Proof: 1) Since for any tree with n>2 vertices not all the vertices are leaves then it is obviously S < V(G) and $\sum_{v_i \in S}|v_i| < O_N$.
2) According to neighbors of the non-leaf vertices there are two cases
Case1: If for each non-leaf vertex has more than one leaf vertex as neighbors
$\sum_{v_i \in S}|v_i| = \gamma_{CNMD}$
Case2: If there exist at least one non-leaf vertex has less than two leaf vertices then
$\exists\, v \in CNMD \text{ and } v \notin S$ then $S \subset CNMD$ which means that $\sum_{v_i \in S}|v_i| < \gamma_{CNMD}$
From 1 and 2 we obtain $\sum_{v_i \in S}|v_i| \leq \gamma_{CNMD} < O_N$.

Example. In the graph 4a below
$\sum_{v_i \in S}|v_i|$=*0.6667*, $CNMD = \{A, C, F, G, D\} \Rightarrow \gamma_{CNMD} = 0.833$ *and*
$O_N = 1.46667 \Rightarrow \sum_{v_i \in S}|v_i| < \gamma_{CNMD} < O_N$.

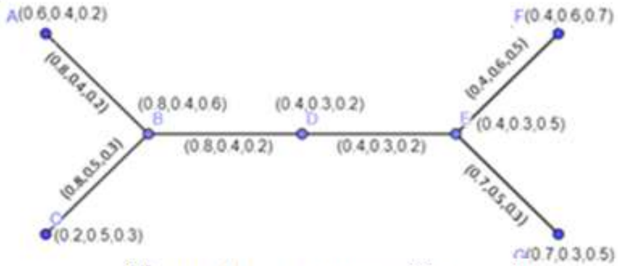

Figure 4a co-neutrosphic graph with vertex without leaf

And in 4b, since all the non-leaf vertices adjacent to a leaf vertex then S= $CNMD \Rightarrow \sum_{v_i \in S}|v_i| = \gamma_{\ CNMD} < O_N$

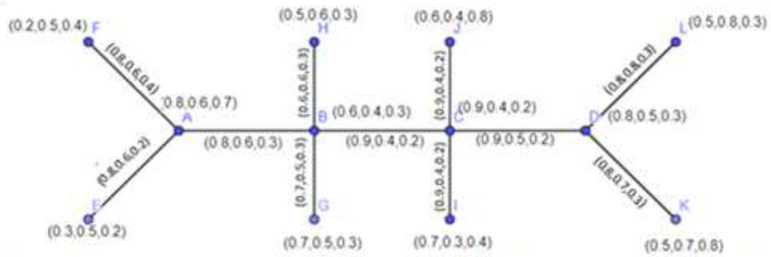

Figure 4b co-neutrosophic graph which all its vertices has leaf

Theorem 3.1. *For any graph G_{CN} ,$\sum_{v_i \in S}|v_i| < \gamma_{\ CNMD} < O_N$. Where S is a set of all the vertices with one or no neighbors.*

Proof: Let G_{CN} be any co-neutrosophic graph,
$\mathcal{D}$ be a CNMD set of G_{CN} and $|\mathcal{D}|= \gamma_{CNMD}(G_{CN})$ *and* $V(G_{CN}) = V(H) \cup V(S)$, S is a set including each of the vertices that has less than two neighbors and H containing vertices of $V(G_{CN})$ which have two or more neighbors. Now we must demonstrate that $S \in \mathcal{D}$ for lower bounded.
Assuming $u \in S$, u is either has a single neighborhood or it is isolated vertex. We terminate that u belong to each MCNMD set of G_{CN} in both situations. Hence $S \in \mathcal{D} \rightarrow |S| \leq |D|$

Furthermore $\sum_{v_i \in S}|u_i| \leq \gamma_{CNMD}$. however, for upper bound $\gamma_{CNMD} < O_N$ is obviously. Hence,

$\sum_{v_i \in S}|v_i| < \gamma_{\ CNMD} < O_N$.

Example3. 3. Consider G_{CN} = (A, B) in figure 5.

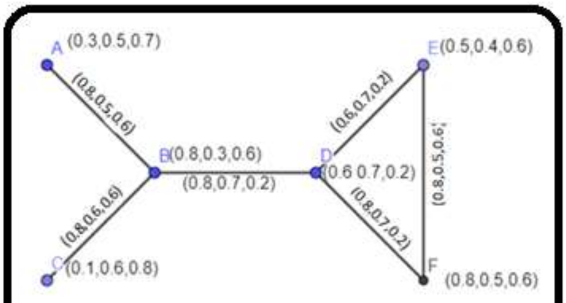

Figure 5

The MCNMD sets are $\mathcal{D}_1$ = {A, D, E, C}, $\mathcal{D}_2$ = {A, D, F, C}, S= {A, C}
$\mathcal{D}_3$ = {A, E, F, C},
$|A| = \left|\frac{1+0.3+0.5-0.7}{3}\right| = 0.36667$, $|B| = \left|\frac{1+0.8+0.3-0.6}{3}\right| = 0.5$, $|C| = \left|\frac{1+0.1+0.6-0.8}{3}\right| = 0.3$
$|D| = \left|\frac{1+0.6+0.7-0.2}{3}\right| = 0.7$, $|E| = \left|\frac{1+0.5+0.4-0.6}{3}\right| = 0.43333$, $|F| = \left|\frac{1+0.8+0.5-0.6}{3}\right| =$ 0.56667

$$|\mathcal{D}_1| = 1.8 , |\mathcal{D}_2| = 1.93334 , |\mathcal{D}_3| = 2.0667 \ |\mathrm{S}| = 0.66667$$

We observe that every MCNMD set contains one neighbor for each vertex.

Hence $\gamma 2AF\ (GAF) = max\ \{|\ \mathcal{D}_1|, |\ \mathcal{D}_2|, |\ \mathcal{D}_3|\} = max\ \{1.8, 1.93334, 2.0667\} = 2.0667 > |S\ |, where\ S = \{\mathrm{A}, \mathrm{C}\}$.

Theorem. 3.2. If D is a CNMD set of G_{CN}, so $V - \mathcal{D}$ is not always CNMD set of G_{CN}.
Proof: Assume that $u \in$ (GCN) and D be the CNMD set of G_{CN}
Case 1 If u have less than two neighbors in GCN, u must belong to every MCNMD set in G_{CN}, Consequently, $V - \mathcal{D}$ is not CNMD because it either has one neighbor who is u or none at all. group of G_{CN}.
Case 2: Assume that each $x \in D$ is dominated by not less than two vertices $y, z \in V - D$. In this situation, each $x \in D$ has at least couple of neighbors in $V - \mathcal{D}$. If D is a CNMD set of G_{CN}, the outcome is attained by cases 1 and 2. Therefore, $V - \mathcal{D}$ not necessary be a CNMD set of G_{CN}.

Example: In the figure (5), D= {A, D, E, C} is MCNMD set but V-D = {B, F} is not CNMD set
Proposition 3.7. *For any co-neutrosophic* G_{CN}= (A, B), γ_{CNMD} (G_{CN}) +γ_{CNMD} ($\overline{G_{CN}}$) ≤ 2 O_N
γ_{CNMD} ($\overline{G_{CN}}$) *is* CNMD *number of complements* G_{CN} .
Proof: Since both of G_{CN} *and* $\overline{G_{CN}}$ *are co-neutrosophic graphs then by theorem3.1*
$\gamma_{\ CNMD}$(GCN) $< O_N$ *and* $\gamma_{\ CNMD}(\overline{GCN}) < O_N$ *then* $\gamma_{\ CNMD}(G_{CN})+ \gamma_{\ CNMD}(\overline{G_{CN}}) < 2O_N$□

Theorem 3.3. Let G_{CN} be co-neutrosophic graph, D⊂ $V(G_{CN}$ *) is a* CNMD *set* of G_{CN} = (A, B) if and only if D *be a multi-dominating (*MD) *set of* G_{CN}^* *and for e=xy is an effective edge in E.*

Proof: Let G_{CN} = (A, B) be co-neutrosophic graph, and $\mathcal{D}$ is CNMD set of G_{CN}, then $x \in V - \mathcal{D}$ has no less than couple of neighbors in D for each vertex, I. e. there exist $y1$, $y2 \in \mathcal{D}$ such that both of $(x, y1)$ and $(x, y2)$ are effective edge and the x is adjacent to both of $y1$, $y2 \in \mathcal{D}$, which means that D is MD of G_{CN}^*.

Let $\mathcal{D}$ be a *MD* set of G_{CN}^* so ∀ y ∈ $V - \mathcal{D}$ there exist pair of vertices $x_{1,}x_2 \in \mathcal{D}$, such that $(x_{1,}, y \in E$ and $(x_{2,}, y) \in E$, since $(x_{1,}, y)$ *and* $(x_{2,}, y)$ are effective edge by the hypothesis, so y adjacent to two vertices in $\mathcal{D}$ with effective edges. Hence, $\mathcal{D}$ is CNMD set of G_{CN}.

Remark. if D be *MD* set of G_{CN}^* it is not necessary D be CNMD set
The example below illustrates the remark

Example 3.4. The theorem 3.3 is not always true unless the edges are not effective. Notice the co-neutrosophic graph G_{CN} in the figure bellow D= {b, c} is

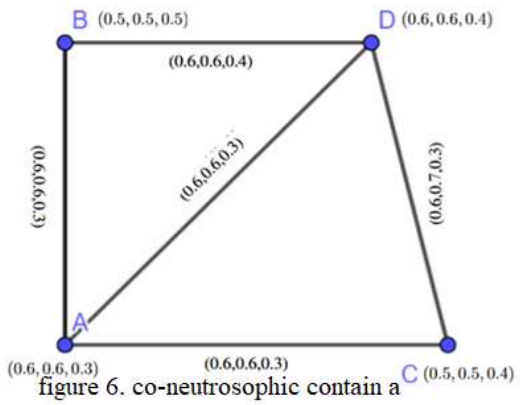

figure 6. co-neutrosophic contain a MCNMD in G_{CN}^* but it isn't in G_{CN}

because the edge cd is not effective so that the vertex $c \in V - D$ is adjacent to just one vertex $b \in D$

Theorem 3.4. Assuming that G_{CN} be a unimodal co-neutrosophic graph with no isolated vertex, *and* $\mathcal{D}$ *is a* γ_{CNM}- set ofG_{CN} , which it is not independent set (IS), then $\gamma_{CN} + t \leq \gamma_{CNMD}$,where $t = \left|\left(T_A(x), I_A(x), F_A(x)\right)\right|$,$x \in \mathcal{D}$

Proof: Let $\mathcal{D}$ be CNMD where $|\mathcal{D}| = \gamma_{CNM}$ (G_{CN}) is not (IS), and $x \in \mathcal{D}$ where $t = \left|\left(T_A(x), I_A(x), F_A(x)\right)\right|$. We have pair of cases:

Case1: If $(x) \cap (V - \mathcal{D}) = \emptyset$, Given that D is not (IS) and G_{CN} has no isolates vertex, $N(x) \cap \mathcal{D} \neq \emptyset$. Thus $\mathcal{D}$ −{x} is CND set of G_{CN}, therefore $\gamma_{CN} \leq |\mathcal{D} - \{x\}| = \gamma_{CNM} - t$
Hence, $\gamma_{CN} + t \leq \gamma_{CNM}$.

Case 2: If $(V - \mathcal{D}) \cap (x) \neq \emptyset$, then $\forall y \in (V - \mathcal{D}) \cap (x)$ since $\mathcal{D}$ is CNMD, ∃ $z \in \mathcal{D}$ where (y, z) is an effective edge, because D is CNMD.D is not independent, hence some of D's vertices are neighbors of z. thus $\mathcal{D} - \{z\}$ is also CND set of G_{CN} ,therefore,

$\gamma_{CN} \leq |\mathcal{D} - \{z\}| = \gamma_{CNM} - t$, hence $\gamma_{CN} + t \leq \gamma_{CNM}$.

Theorem 3.5. Every co-neutrosophic vertex cover set is a CNMD set of G_{CN} if G_{CN} is any co-neutrosophic graph where each vertex has no less than pair of neighbors. *Further* $\gamma_{CNM} = \alpha_o$.

Proof: Suppose that A is a minimum co-neutrosophic vertex covers set of G_{CN}, and

$y \in V(G_{CN}) - A$.

Obviously, $(y) \in A$. Since each vertex in G_{CN} has more than one neighbor, the vertex y has at least two neighbors in A. This implies that A is CNMD set of G_{CN}. Hence, γ_{CNM}= α0.

Example: Let G_{CN} be co-neutrosophic graph as follows:
the co-neutrosophic vertex cover sets of G_{CN} are $C_1 = \{A, C, E\}, C_2 = \{B, D\}$
then co-neutrosophic vertex covering number (α_O)=$|C_2|$, but C_2 is also MCNMD set then $\gamma_{CNM}(G_{CN}) = |C_2|$

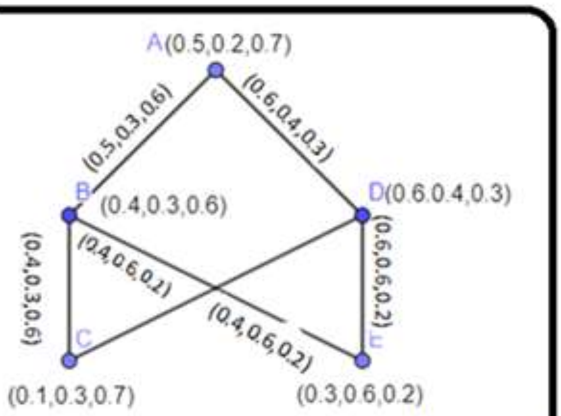

Figure 7co-neutrosophic graph

Corollary 3.5. Let G_{CN} be co-neutrosophic graph. If $\gamma_{CNM}(G_{CN}) \neq \alpha_O(G_{CN})$ then G_{CN} involves a vertex has less than two neighbors.

Theorem 3.6. If G_{CN}= (A, B) is a connected a unimodal co-neutrosophic graph and with γ_{CN}=γ_{CNM} ,then each vertex has more than one neighbor.
Proof: let $x \in \mathcal{D}$ be a vertex with a single neighbor, where $\mathcal{D}$ is a MCNMD set of G_{CN} such that $|\mathcal{D}| = \gamma_{CNM}$,if $V - \mathcal{D} = \emptyset$ then $\gamma_{CN} < \gamma_{CNM} < O_N = \gamma_{CNM}$ which is a contradiction, thus $V - \mathcal{D} \neq \emptyset$. Let x has neighbor y. If $y \in \mathcal{D}$, so $\mathcal{D}' = \mathcal{D} - \{x\}$ is CND set of G_{CN} with $|\mathcal{D}'| = |\mathcal{D} - \{x\}|$ this leads to $\gamma_{CN} \neq \gamma_{CNM}$, a contradiction.
If $y \in V - \mathcal{D}$, since $\mathcal{D}$ is MCND set, then $\exists\ z \in N(y) \cap \mathcal{D}$ and $z \neq x$, for all vertex in $V - \mathcal{D}$ has no less than couple of neighbors in $\mathcal{D}$, we noticed that $H = \mathcal{D} - \{z, x\} \cup \{y\}$ is CND set of

GCN with $|H| = |\mathcal{D} - \{z, x\} \cup \{y\}|$ thus $\gamma_{CN} \neq \gamma_{CNM}$ which a contradiction. Hence, each vertex in G_{CN} has no less than two neighbors
when $\gamma_{CN} = \gamma_{CNM}$.

Proposition 3.8. Let G_{CN} = (A, B) be any co-neutrosophic graph with vertices each of them has two or more neighbors then $\gamma_{CNM} + \beta_o \leq O_N . where\ |S| = \beta_o$
Proof: Suppose that S is a maximum independent co-neutrosophic set of G_{CN}. then V- S contains all of the neighbors of each vertex of S. Every vertex has two or more neighbors by hypothesis; hence V-S must be a CNMD set of G_{CN}. Thus $\gamma_{CNM} \leq | V - S | = O_{CN} - \beta_o$.
Hence, $\gamma_{CNM} + \beta_o \leq O_{CN}$.

Proposition 3.9. If G_{CN} = (A, B) be a co-neutrosophic graph with S being the only maximal independent co-neutrosophic set, then $\gamma_{CNM} \leq \beta 0$.Where $|S| = \beta_o$
Proof: assume S be a single maximal co-neutrosophic independent set of G_{CN}, suppose that $\exists\ x \in V - S$. if x has no neighbor then x has to be in S. therefore, x is neighbor with only one vertex $y \in S$, then $S - \{y\} \cup \{x\}$ is the second maximal co-neutrosophic independent set of GCN. This results in a conflict with S. Hence, $\gamma_{CNM} \leq |S| = \beta_o$

4.Inverse of (CNMD) in G_{CN}

Definition 4.1. Let G_{CN} be any co-neutrosophic graph without isolated vertex and D_{MCN} be minimum co-neutrosophic multi-domination of G_{CN} if $V - D_{MCN}$ contains a (CNMD) D_{MCN}^{-1} then D_{MCN}^{-1} is called invers (CNMD)s of G_{CN} with respect to D_{MCN}

Remark4.1. An inverse (CNMD) $D_{MCN}^{-1}\ of\ G_{CN}$ is said to be a minimal if no proper subset of D_{MCN}^{-1} is inverse CNMD of G_{CN}

Definition 4.2. The minimum set among all inverse CNMD sets is said to be a minimum inverse CNMD of G_{CN},and γ_{CNM}^{-1} of G_{CN} is the term used to describe the maximum neutrosophic cardinality taken over all minimum inverse CNMD sets of G_{CN}. and denoted by $\gamma_{CNM}^{-1}(G_{CN})$ or simply γ_{CNM}^{-1}

Remark 4.2. A minimum inverse CNMD of G_{CN} has a maximum neutrosophic cardinality is called $\gamma_{MCNM}^{-1} - set$ of G_{CN}

Example 4.1. Consider a co-neutrosophic graph G_{CN}, which is given in figure 8

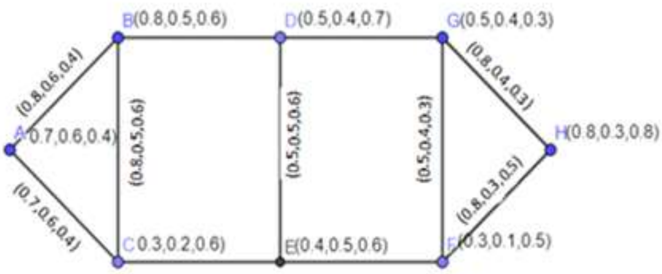

Figure 8 co-neutrosophic graph

MCNMD sets of G_{CN} are $D_{CNM(1)} = \{B, G, C, F\}, D_{CNM(2)} = \{A, H, D, E\}$

$\gamma_{CNM} = \max\{\|D_{CNM(1)}\|, \|D_{CNM(1)}\|\}$=max $\{\sum_{v\in D_{CNM(1)}}|v|, \sum_{u\in D_{CNM(2)}}|u|\} = \max\{1.7,1.9\} = 1.9$

$D_{CNM(1)}$ is inverse CNMD set $\gamma^{-1}_{CNM} = 1.7$

Theorem 4.1.

Let G_{CN} be any co-neutrosophic graph, then D^{-1}_{MCN} the CNMD inverse of G_{CN} is aminimal if and only if for each vertex $x \in D^{-1}_{MCN}$ either

1) $|N(x) \cap D^{-1}_{MCN}| < 2\ or$
2) $\exists\ y \in V - D^{-1}_{MCN}$ where $\{N(y) \cap D^{-1}_{MCN}\} = 2\ and\ y \in N(x)$.

   Proof: Let D^{-1}_{MCN} be $\gamma^{-1}_{CNM} - set$ of G_{CN}, Suppose the aforementioned condition is not satisfied. $\exists\ x \in D^{-1}_{MCN}$ where $|N(x) \cap D^{-1}_{MCN}| \geq 2$ and for each vertex $y \in V - D^{-1}_{MCN}$ either $|N(y) \cap D^{-1}_{MCN}| > 2\ or\ y \notin N(x)$

   Consider $D'_{MCN} = D^{-1}_{MCN} - \{x\}$,since x has at least two neighbors in D^{-1}_{MCN} thus D'_{MCN} is inverse CNMD of G_{CN}, an opposition to minimalism D^{-1}_{MCN}.

Conversely, if D^{-1}_{MCN} is an inverse CNMD of G_{CN} that satisfies (1) and (2), then consider $D'_{MCN} = D^{-1}_{MCN} - \{x\}$ for any vertex $x \in D^{-1}_{MCN}$. D'_{MCN} is not inverse CNMD if condition (1) holds, and it is not inverse CNMD if condition (2) holds if D'_{MCN} has one neighbor named y. Hence D^{-1}_{MCN} is minimal inverse CNMD set of G_{CN}.

Proposition 4.1. If G_{CN} be any co-neutrosophic graphs contains at least one vertex has no more than one neighbor then inverse CNMD not exist

Proof: Consider G_{CN} be any co-neutrosophic graph and $x \in V(G_{CN})$ has at most one neighbor,i.e. $|N(X)| \leq 1$. Then,x *contained in* every MCNMD of G_{CN} i.e.

$x \notin V - D_{CNM}$,where D_{CNM} is MCNMD of G_{CN}.suppose $D_{CNM}^{-1} \subseteq V - D_{CNM}$ is inverse D_{CNM} of G_{CN} with respect to D_{CNM}. Since $x \in CNMD$ then D_{CNM}^{-1} must has two neighbor of x .this is not possible, by $|N(x)| \leq 1$. *Hence* G_{CN} has no inverse CNMD

Corollary 4.1. if $G_{CN} \cong P_n^{CN}$ or G_{CN} ≅strong co-neutrosophic star graph then an inverse CNMD set not exist.

Observation: If there is inverse CNMD of G_{CN}, then not necessary in general $\gamma_{CNM} \geq \gamma_{CNM}^{-1}$

i) For any co-neutrosophic graph G_{CN} does not contain isolated vertices, every invers CNMD is inverse neutrosophic dominating set.

ii) For any co-neutrosophic graph G_{CN} has inverse CNMD then $[A(x) + A(y)] \leq \gamma_{CNM}^{-1} \leq \sum |u_i|$ where $x, y \in D_{CNM}^{-1}$ *and* $u_i \in V - D_{CNM}$.

Theorem 4.2. For any co-neutrosophic graph G_{CN} has $CNMD^{-1}$ then a vertex $x \in V - D_{CNM}$ belong to each $CNMD^{-1}$ of G_{CN} in the event that x has two or three neighbors.

Proof: Let D_{CNM} be a MCNMD and D_{CNM}^{-1} be an inverse CNMD of G_{CN}. Then, each vertex $x \in V - D_{CNM}$ has at more than one neighbor in D_{CNM} . Then, there exist two cases.

Case(I) Suppose that $x \in V - D_{CNM}$ and x has exactly couple of neighbors say $\{y, z\}$,i.e.

$N(x) = \{y, z\}$, since D_{CNM} is MCNMD of G_{CN} then $\{y, z\} \in D_{CNM}$ therefore $N(x) - \{y, z\}$=∅ , so x has no other neighbors in V which dominates. Therefore, x needs to dominated by itself. As a result, x is contained in every inverse CNMD.

Case(ii) Suppose that $x \in V - D_{CNM}$ has precisely three neighbors in G_{CN}. Let y, z and r are three neighbors of x, i.e. $N(x) = \{y, z, r\}$, since $x \in V - D_{CNM}$ and D_{CNM} is MCNMD.Then x has at least two neighbors in D_{CNM}, consider y and z are two neigbors in D_{CNM} *of* x, which dominates x. Now $N(x) - \{y, z\}$=$\{r\}$, i.e. remaining a singular neighbor of x in $V - D_{CNM}$ since $D_{CNM}^{-1} \subseteq V - D_{CNM}$.If $x \notin D_{CNM}^{-1}$, then D_{CNM}^{-1} must have at least two neighbors of x ,but x has only one neighbor in $V - D_{CNM}$. Therefore ,x belong to every D_{CNM}^{-1} of G_{CN}.

Proposition 4.2. Let $G_{CN} = (A, B)$ be any connected co-neutrosophic graph on

$G_{CN}^* = (V, E)$; then $|V| > 3$ if D_{CNM}^{-1} exists,

Proof: Since is connected and D^{-1}_{CNM} exists by hypothesis, then

1) $\exists\, x, y \in D^{-1}_{CNM}$ *and then* $x, y \in V - D_{MCNM}$
2) G_{CN} *has* D_{MCNM} then $\exists\, z, f \in D^{-1}_{CNM}$ *and then* $z, f \in V$
   From (1) and (2) $x, y, z, f \in V - D_{MCNM} \cup D_{MCNM} = V$
   Which is the proof.

Proposition 4.3. $G_{CN} = (A, B)$ be co-neutrosophic graph on

$G^*_{CN} = (V, E)$if there is inverse CNMD of G_{CN} then $\gamma_{CNM} + \gamma^{-1}_{CNM} \le |V|$

Proof: Let D_{CNM} and D^{-1}_{CNM} be a MCNMD and invers CNMD of G_{CN} respectively.

then either G_{CN} be connected or not

1) For every element $x \in D_{CNM} \Rightarrow x \epsilon V$ $i.e. D_{CNM} \subset V$
2) For every element $y \in D^{-1}_{CNM} \Rightarrow y \epsilon V - D_{CNM}$ $i.e. D_{CNM} \subset V - D_{CNM} \subset V$
   from (1) and (2) $D_{CNM} \cup D^{-1}_{CNM} \subset V$
   then $|D_{CNM}|+|D^{-1}_{CNM}|<|V| \Rightarrow \gamma_{CNM} + \gamma^{-1}_{CNM} \le |V|$.

Proposition 4.4. Let $G_{CN} \cong k^{CN}_n$ is complete co-neutrosophic graph with n≥ 4 vertices,

$\gamma^{-1}_{CNM}(k^{CN}_n) = \max[\varphi(x) + \varphi(y)], x, y \in V(k^{CN}_n) - CNMD.$

Proof: Given $G_{CN} \cong k^{CN}_n$ and D_{CNM} be a MCNMD of k^{CN}_n. Then, $CNMD$ contain two vertices with maximum neutrosophic value by preposition (4.2.27) thus $< V - D_{CNM} >$ is k^{CN}_{n-2}. Then, the inverse CNMD OF k^{CN}_n is MCNMD OF k^{CN}_{n-2}.Hence

$\gamma^{-1}_{CNM}(k^{CN}_n) = \gamma_{CNM}(k^{CN}_{n-2}) = \max[\varphi(x) + \varphi(y)]\, x, y \in V(k^{CN}_n) - CNMD$, the proof is complete.

Proposition 4.5. Let $G_{CN} \cong C^{CN}_n$ with $n \ge 2k$ vertices, k≥ 2 and each edge is an effective then

$\gamma^{-1}_{CNM} = \min\{\sum_{i=0}^{\frac{n}{2}-1} A(x_{j+2i}); j = 1,2\}$,where $A(x_{j+2i}) = (T_A(x_{j+2i}), I_A(x_{j+2i}), F_A(x_{j+2i}))$

Proof: Consider $C^{CN}_n = \{x_1, x_2, \dots, x_n\}$ be cycle with even vertices by Proposition 3.6, there are two MCNMD sets $\{D_{2A(1)}, D_{2A(2)}\}$ of C^{CN}_n and one of them has maximum neutrosophic cardinality is CNMD number of C^{CN}_n ,

i.e., if $\|D_{2A(1)}\| > \|D_{2A(2)}\|$, then $D_{2A(2)}$ is inverse CNMD of C^{CN}_n and

$\gamma_{CNM}^{-1}(k_n^{CN}) = \|D_{2A(2)}\|$

similarly, if $\|D_{2A(2)}\| > \|D_{2A(1)}\|$ then $\gamma_{CNM}^{-1}(k_n^{CN}) = \|D_{2A(1)}\|$

proposition 4.6. If $G_{CN} \cong k_{n,m}^{CN} = (A, B)$ is a complete co-neutrosophic graph on bipartite underline graph $G_{CN}^* = K_{n\times m} = (V, E)$ then

$$\gamma_{CNM}^{-1}\left(k_{n,m}^{CN}\right) = \left\{\begin{matrix} either\ Sc(X) or\ Sc(y) & if\ 2 \le n, m \le 3\ and\ m \ne n \\ Min\{Sc(X), Sc(Y)\} & n = m \le 3 \\ Min\{Sc(X), Sc(Y), Sc\left(x_i + x_j + y_i + y_j\right)\} & m, n > 3 \\ x_i, x_J \in X - D_{CNM}\ and\ y_i, y_J \in Y - D_{CNM} & \end{matrix}\right\}$$

Where Score(X) is Sc(X)=$\frac{1+\sum_{v_i\in X} T_A(v_i)+\sum_{v_i\in X} I_A(v_i)-\sum_{v_i\in X} F_A(v_i)}{3}$ and

Sc(Y)=$\frac{1+\sum_{v_j\in Y} T_A(v_j)+\sum_{v_j\in Y} I_A(v_j)-\sum_{v_j\in Y} F_A(v_j)}{3}$

Proof: Since G_{CN}^* is bipartite complete graph then $V = X \cup Y, where\ X, Y$ are sets of independent(non-adjacent) vertices, such that for each $x \in X \ \exists\ edges\ e_j = xy_j, y_j \in Y, J = 1,2, \dots m$

Since G_{CN} is complete co-neutrosophic graph over G_{CN}^* then for each edge $e \in E\ is\ an\ effective\ edge$

now let D_{CNM} is MCNMD of $k_{n,m}^{CN}$ there three cases:

Case 1: $if n = 2\ and = 3$, then obviously X is D_{CNM} and Y is $D_{CNM}^{-1} \Rightarrow \gamma_{CNM}^{-1}\left(k_{n,m}^{CN}\right) = Sc(Y)$, in the same way we can proof that $\gamma_{CNM}^{-1}\left(k_{n,m}^{CN}\right) = Sc(X)\ if\ m = 2$.

If n=2 then m≥ 3 it is obvouisly X is MCNMD of $k_{n,m}^{CN}$ because there exist not any subset of Y can be dominating set due to the independently of Y which means that Y is D_{CNM}^{-1} and $\gamma_{CNM}^{-1} = Sc(Y)$ .Similarly in case of m=2 and n=3

Case 2: $If n = m \le 3$, then if $n = m = 2$ then both of X and Y can be D_{CNM} and conversely both of X and Y can be $D_{CNM}^{-1} \Rightarrow \gamma_{CNM}^{-1}\left(k_{n,m}^{CN}\right) = \min(\,Sc(x), Sc(Y)\,)$.

The same condition satisfies if m=n=3

Case 2: $If n, m > 3$, then D_{CNM} contain at least four vertices in three different situations

i) D_{CNM} contain two vertices from each of X and Y and conversely D_{CNM}^{-1} also contains two vertices from each of X and Y
ii) X contain only four vertices and equal to D_{CNM} then Y is equal to D_{CNM}^{-1}
iii) Y contain only four vertices equal to D_{CNM} then X is equal to D_{CNM}^{-1}
iv) Both of X and Y contains four vertices then one of them is equal to D_{CNM} and the other is equal to D_{CNM}^{-1} and vice versa.

i.e. $\underset{x_i, x_J \in X - D_{CNM} \text{ and } y_i, y_J \in Y - D_{CNM}}{Min}\{Sc(X), Sc(Y), Sc\big(x_i + x_j + y_i + y_j\big)\}$

that is the proof.

Proposition 4.7. Every invers CNMD of G_{CN} is invers multi-dominating set of crisp graphs G_{CN}^*

Proof: Let D_{CNM}^{-1} be an inverse CNMD of G_{CN} , then $\forall\, y \in V - D_{CNM}^{-1}$ has two or more neighbors in D_{CNM}^{-1} ,i.e. there exist $x_1, x_2 \in D_{CNM}^{-1}$ such that $\omega(y, x_1) = A(x_1) \vee A(y) > 0$ and $\omega(y, x_2) = A(x_2) \vee A(y) > 0$ it implies that (y, x_1) and $(y, x_2) \in \omega^*$.

Therefore, D_{CNM}^{-1} contains two neighbors of y. Hence D_{CNM}^{-1} is inverse multi-dominating set of G_{CN}^*.

Remark. The converse of preposition (4.7) is not always true. It is illustrated in the example that follows.

Example. Given G_{CN} and G_{CN}^* in figure 9 (a), (b) respectively, obviously {E, C} is inverse multi-dominating set of G_{CN}^* but not inverse in G_{CN}.

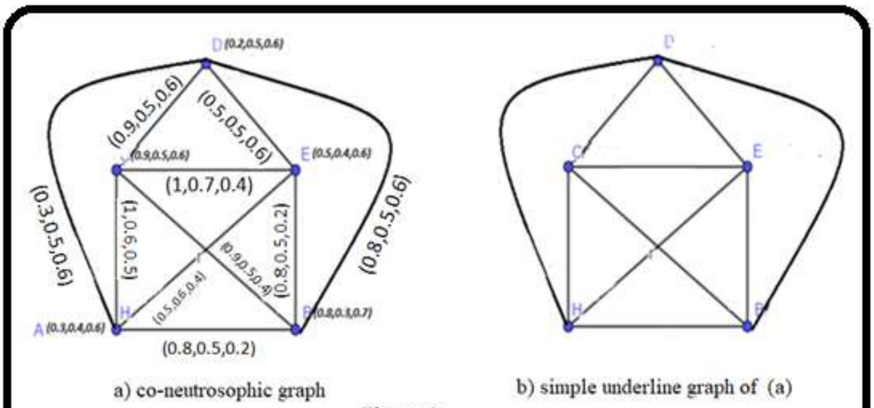

Figure :9

Proposition 4.8. Let G_{CN} be co-neutrosophic graph.Inverse multi dominating set of G^*_{CN} is inverse $CNMD^{-1}$ of G_{CN} If $e = xy$ is an effective edge $\forall\ (x, y) \in E(G_{CN})$.

Proof: Let D^{-1}_M be a γ^{-1}_M set of G^*_{CN} . then, $\forall\ y \in V - D^{-1}_M\ \exists\ x_1, x_2 \in D^{-1}_M$ such that $(y, x_1), (y, x_1) \in E$.

Since each edge in G_{CN} is effective thus D^{-1}_M has two effective neighbors of y. Hence, D^{-1}_M is inverse multi dominating set of $\mathrm{G_{CN}}$.

Conclusion. In this paper, the concepts of a (CNMD) set and number in a co-neutrosophic graph ($\mathrm{G_{CN}}$) introduced and examined also invers multi-dominating setD^{-1}_{CNM} and its number are the purpose of this essay. The co-neutrosophic multi-domination number (CNMD) determined for particular classes of ($\mathrm{G_{CN}}$) and deduce restrictions on the accompanying (CNMD number).

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