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Comparative Analysis between EWMA, DEWMA and Mixed Tukey EWMA Control Chart

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Abstract

The control chart is a good tool for monitoring. Single and double exponential weighted moving average control charts are suitable charts to detect small shifts in the process parameters. For good detection, an ability the displaced moving average chart relies depends on the normality assumption. Occasionally, there is a need to see small trends instead of shifts. A double exponentially weighted moving average (DEWMA) control chart extends the EWMA control chart by performing exponential smoothing twice. In recent years, it has been seen that the DEWMA control chart is more efficient than the EWMA control technique for detecting small shifts in the process mean. With technological advancement, we improve strategies that combine many elements into a single framework. This paper is a development of a previous effort to build a better charting framework in the form of a mixed Tukey's control chart. Tukey's control charts are a good choice for robust process monitoring. According to the comparison study, the suggested scheme is an effective rival to EWMA. We build improved procedures that include several features in a single construction with technological innovation. This paper aims to create a better chart and provide a comparative analysis between EWMA, DEWMA, and mixed Tukey EWMA, to show the best process control chart. We improve strategies that combine many elements into a single framework. Practically, we have implemented the proposed charts for data sets, related to the chemical components of water.

Keywords: Control Chart, Exponentially Weighted Moving Average Control Chart, Double Exponential Weighted Moving Average Control Chart, Tukey's Control Chart, Average Run Length.

تحليل مقارنة بين لوحات السيطرة (EWMA) و (DEWMA) و (Mixed Tukey EWMA)

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الخلاصة

تعد لوحات السيطرة النوعية أداة جيدة لمراقبة العمليات، وعلية فأن لوحات السيطرة لـ (EWMA) و لوحات السيطرة لـ (DEWMA) المزدوجة لوحات مناسبة لاكتشاف التحولات الصغيرة في معلمات العملية. ولآكتشاف تلك التحولات الصغيرة بشكل جيد، نفترض ان المخطط في حالتها الطبيعية. ويعمل لوحات السيطرة الموزونة أسيا والمزدوجة (DEWMA) على توسيع مخطط التحكم (EWMA) عن طريق إجراء تجانس أسي مرتين. في السنوات الأخيرة ، لوحظ أن مخطط السيطرة (DEWMA) أكثر كفاءة من تقنية لوحة EWMA لاكتشاف التحولات الصغيرة في متوسط العملية. هذا البحث عبارة عن تطوير لجهد سابق لبناء إطارعمل تخطيطي أفضل في شكل لوحة السيطرة Mix Tukey EWMA. وتعد مخطط Mix Tukey خيارًا جيدًا لمراقبة العمليات الشاقة. ووفقا لدراسة المقارنة ، يعد المخطط المقترح منافسًا فعالًا لـ EWMA. لذا يهدف هذا البحث على إنشاء مخطط أفضل وتحليل مقارنة بين EWMA و DEWMA و EWMA و Mix Tukey ، لإظهار أفضل مخطط للتحكم في العملية. ونقوم بتحسين الاستراتيجيات التي تجمع بين العديد من العناصر في إطار واحد. وفي الجانب العملي قمنا بتنفيذ المخططات المقترحة على البيانات المتعلقة بالمكونات الكيميائية للمياه.

الكلمات المفتاحية: السيطرة النوعية، لوحات الاوساط المتحركة الموزونة، لوحات الاوساط المتحركة الموزونة المزدوجة، لوحة Mixed Tukey ، متوسط طول التشغيل.

Introduction

Roberts' EWMA control chart, introduced in 1959, effectively detects small shifts. The process distribution is assumed to be normal in these control charts. In many situations, the process distribution is non normal. The EWMA control chart statistics are given where k represents the times' number of the standard deviation and is selected for a given lambda, such as the average run length under in-control $ARL0 \cong 370$ **[1].** For example, Crowder (1987) shows that for λ =0.1 and k = 2.7, the $ARL0$ ≈370. Practically, the EWMA control chart is easy to use.

Exponentially Weighted Moving Average Control Chart (EWMA).

The exponentially weighted moving average is defined as:

$$Z_i = \lambda x_i + (1-\lambda) z_{i-1} \quad \text{......} (1)$$

Where:

$$Z_0 = \mu_0$$
$$Z_i = \lambda x_i + (1-\lambda)[\lambda x_{i-1} + (1-\lambda) z_{i-2}$$
$$= \lambda x_i + \lambda(1-\lambda) x_{i-1} + (1-\lambda)^2 z_{i-2} \quad \text{......} (2)$$

For z_{i-j} , $j = 2,3, \dots . t$, we obtain that

$$Z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j x_{i-j} + (1-\lambda)^i z_0 \quad \text{......} (3)$$

The weights $(1-\lambda)^i$ decrease geometrically with the age of the mean. Furthermore, the weights sum to unity, since:

$$\lambda \sum_{j=0}^{i-1} (1-\lambda)^j = \lambda \left[\frac{1-(1-\lambda)^i}{1-(1-\lambda)}\right] = 1-(1-\lambda)^i \quad \text{......} (4)$$

t is not sensitive to the assumption of normality. As a result, it makes for the perfect control chart for use with individual observations. If the observations X_i are independent random variables with variance S^2, then the variance of Z_i is:

$$\sigma_{zi}^2 = \sigma^2 \left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right] \quad \text{......} (5)$$

The EWMA control chart is drawn by value of z_i against the sample i (or time). Therefore, the EWMA control chart's control limits and center line are as follows:

$$\begin{aligned} \text{UCL} &= \mu_0 + L\sigma\sqrt{\left(\frac{\lambda}{2-\lambda}\right)[1-(1-\lambda)^{2i}]} \\ \text{CL} &= \mu_0 \\ \text{UCL} &= \mu_0 - L\sigma\sqrt{\left(\frac{\lambda}{2-\lambda}\right)[1-(1-\lambda)^{2i}]} \end{aligned} \quad \text{.......... (6)}$$

Note that the term $\left[1-(1-\lambda)^{2i}\right]$ in these equations approaches unity as i get larger, the control limits are given by the following [1],[3],[10],[12]:

$$\begin{aligned} UCL &= \mu_0 + L\sigma\sqrt{\left(\frac{\lambda}{2-\lambda}\right)} \\ CL &= \mu_0 \\ LCL &= \mu_0 - L\sigma\sqrt{\left(\frac{\lambda}{2-\lambda}\right)} \end{aligned} \quad \text{.......... (7)}$$

Double Exponentially Weighted Moving Average Control Chart

The DEWMA control chart extends the EWMA control chart by conducting exponential smoothing twice. Shamma was the first to construct a (DEWMA) control chart (1992). Zhang and Chen (2005) proposed a DEWMA approach as an extension of the (EWMA) technique. The monitoring process mean based on average run length (ARL) and analyzing the DEWMA control chart's signal resistance measure, the process means were developed under the assumption of normality.

The individuals for the double exponentially weighted moving average are:

$$Z'_i = \lambda Z_i + (1-\lambda)Z'_{i-1} \quad \text{.......... (8)}$$

Where:

$Z_i = \lambda x_i + (1-\lambda)Z_{i-1}$, with its corresponding initial values $Z'_0 = Z_0 = \mu_0$. The control chart is built by plotting the value Z'_i with its limits, using k times the variance of Z'_i. That is, the DEWMA value is plotted with the upper and lower limits versus i.

For $x_i \sim N(\mu_0, \sigma)$, $\quad i = 1,2,\dots,n$

$$Z_i = \lambda x_i + (1-\lambda)Z_{i-1} \quad \text{.......... 9}$$

$$Z'_i = \lambda Z_i + (1-\lambda)Z'_{i-1} \quad \text{......} (10)$$

where $0 < \lambda < 1$ and $Z'_0 = Z_0 = \mu_0$.

It can be shown that:

$$E(Z'_i) = \mu_0$$

and

$$Var(Z'_i) = \lambda^4 \frac{1+(1-\lambda)^2-(i^2+2i+1)(1-\lambda)^{2i}+(2i^2+2i+1)(1-\lambda)^{2i+2}-i^2(1-\lambda)^{2i+4}}{(1-(1-\lambda)^2)^3}\sigma^2 \quad \text{......} (11)$$

For large values of i the variance becomes:

$$Var_{asymp}(Z'_i) = \frac{\lambda(2-2\lambda-\lambda^2)}{(2-\lambda)^3}\sigma^2 \quad \text{......} (12)$$

The control limits for large value of i become:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}$$

$$CL = \mu_0 \quad \text{......} (13)$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}$$

In the state the value of ARL_0 of the proposed control is given as:

$$ARL_0 = \frac{1}{P_{out}^0} \quad \text{......} (14)$$

$$P_{out}^1 = P\left(Z_i < LCL_1|_{\mu_1}\right) + P\left(Z_i > UCL_1|_{\mu_1}\right)$$

$$= \Phi\left(K_1\sqrt{n} + \frac{\delta\sqrt{n}}{\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}}\right) + 1 - \Phi\left(K_1\sqrt{n} + \frac{\delta\sqrt{n}}{\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}}\right) \quad \text{......} (15)$$

is obtained the probability of repetition for the control chart as:

$$P_{rept,\mu}^1 = P(UCL_2 Z_i < LCL_1|_{\mu_1}) + P\left(UCL_1 Z_i < LCL_2|_{\mu_1}\right)$$

$$= \begin{matrix} \Phi\left(\frac{K_1\sqrt{n}\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}+\delta\sqrt{n}}{\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}}\right) - \Phi\left(\frac{K_2\sqrt{n}\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}+\delta\sqrt{n}}{\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}}\right) \\ \Phi\left(\frac{-K_2\sqrt{n}\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}+\delta\sqrt{n}}{\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}}\right) - \Phi\left(\frac{-K_1\sqrt{n}\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}+\delta\sqrt{n}}{\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}}\right) \end{matrix} \quad \text{......} (16)$$

where Φ(.) is a standard normal cumulative distribution function and the probability of repeat for the proposed control chart is calculated as follows:

$$P^1_{out} = \frac{P^1_{out}}{1-P^1_{rept\ \mu 1}} \quad (17)$$

The ARL_1 out-of-control of the proposed is given as:

$$ARL_1 = \frac{1}{P^1_{out}}$$

According to Brown and Meyer's (1961) explanation of the exponential smoothing theorem, a DEWMA can forecast a linear prediction with a linear relationship using the following equation.:

$$F_{i+t} = a_i + b_i t \quad (18)$$

where F_{i+t} is the forecast in the t period,

$$a_i = 2Z_i - Z_i'$$

And

$$b_i = \frac{\lambda}{1-\lambda}(Z_i - Z_i') \quad (19)$$

F_t is called the statistic of the DEWMABLP. It is possible to create three control charts: first, a control chart for the intercept a_i, second, a control chart for the slope b_i that is used to test that will be similar than the EWMA control chart; if there is a linear drift or trend; and third, a control chart for a linear prediction t periods ahead of i, F_t, that is used to test if there is a linear trend statistically in control or not [5],[7],[8],[9].

The expected value of a_i is the center line for a control chart is given as:

$$E(a_i) = E(2Z_i - Z_i')$$
$$= 2E(Z_i) - E(Z_i') \quad (20)$$
$$= 2\mu_0 - \mu_0 = \mu_0$$

This can be verified using equations of ($E(Z_i) = \mu_0$) and ($E(Z'_i) = \mu_0$).
Using equations of ($Var(Z_i)$) and ($Var(Z'_i)$), the variance of a_i can be obtained as:

$Var_{asym}(a_i) = Var(2Z_i - Z_i')$ According to Brown (1962) the asymptotic variance for a predict value of a_i is:

$$Var_{asym}(a_i) = \frac{\lambda(1+4(1-\lambda)+5(1-\lambda)^2)}{(1+(1-\lambda))^2}\,\sigma^2 \quad \text{.......... (21)}$$

Where the values of i is large then the control limits for the a_i become:

$$UCL = \mu_0 + k\sigma\sqrt{\frac{\lambda(1+4(1-\lambda)+5(1-\lambda)^2)}{(1+(1-\lambda))^2}\,\sigma^2}$$
$$CL = \mu_0 \quad \text{.......... (22)}$$
$$LCL = \mu_0 - k\sigma\sqrt{\frac{\lambda(1+4(1-\lambda)+5(1-\lambda)^2)}{(1+(1-\lambda))^2}\,\sigma^2}$$

Tukey's control chart (TCC)

Tukey (1970) developed the box plot, a well-known statistical tool for detecting patterns, and White and Schroeder (1987) published a chart based on it. Alemi (2004) employed Tukey's Control Chart, a standard version of the box diagram for individual observations with independent and identical distributions. The TCC has the same performance as Shewhart's control chart, but the TCC constructed the control chart with fewer observations. When the number of comments was decreased, the likelihood of a false alert decreased. The Adjusted Tukey's Control Chart (ATCC) and the in-control ARL are used in this work to enhance Tukey's control chart and lower the likelihood of false alarms.

The control limits for TCC are defined as:

$UCL = F^{-1}(0.75) + L \times IQR$, and .... (23)

$LCL = F^{-1}(0.25) - L \times IQR$

where UCL and LCL are upper and lower control limits, respectively [2]. $F^{-1}(0.75)$ and $F^{-1}(0.25)$ are the third quartile (Q_3) and first quartile Q_1 and IQR is the Inter- Quartile Range of Q_1 And Q_3
($IQR = Q_3 - Q_1$), Where the parameter L denotes the coefficient of control limits, that may be fixed, and Q1, Q2, and Q3, respectively, which this value of L is usually (1.5) for the case of a normal distribution assumption [4],[9],[10],

$$LCL = Q_1 - L \times IQR$$
$$CL = Q_2 \quad \text{.......... (24)}$$

$$UCL = Q_3 + L \times IQR$$

TCC-EWMA Control Chart

The plotting statistic for TCC-EWMA is (cf. Khaliq et al., 2016) given as:

$$G_i = Z_i = \lambda X_i + (1-\lambda)Z_{i-1} X_i + (1-\lambda)G_{i-1} \quad (25)$$

The variance of TCC-EWMA statistic is as follows:

$$Var(Z_i) = \frac{\widehat{IQR}\lambda(1-(1-\lambda)^{2i})}{2-\lambda} \quad (26)$$

Z_i is initially set to zero, $Z_0 = \mu_0$, and λ is the smoothing parameter,

Therefore; the control limits, are given as:

$$\begin{aligned} UCL &= Q_3 + L_i\widehat{IQR}\sqrt{\frac{\lambda(2-2\lambda+\lambda)^{2i}}{2-\lambda}} \\ CL &= Q_2 \\ LCL &= Q_1 - L_i\widehat{IQR}\sqrt{\frac{\lambda(2-2\lambda+\lambda)^{2i}}{2-\lambda}} \end{aligned} \quad (27)$$

Where Q_1, Q_3, and $\widehat{IQR}$ denote the first, third, and interquartile ranges, respectively, and L denotes the control limit coefficient, determined by the pre-specified false alarm rate or ARL_0. EWMA-steady-state TCC's control limitations are as follows [10],[11]:

$$\begin{aligned} UCL &= Q_3 + L_i\widehat{IQR}\sqrt{\frac{\lambda}{2-\lambda}} \\ CL &= Q_2 \\ LCL &= Q_1 - L_i\widehat{IQR}\sqrt{\frac{\lambda}{2-\lambda}} \end{aligned} \quad (28)$$

(When $(i \to \infty, then\ (1-\lambda)^{2i} \to 0)$)

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Numerical Application

This paper used the actual data of one chemical component of water analysis the Total Dissolved Solid (TDS) with a sample size of (50), are shown in the table (1) which contain the Values of (EWMA, DEWMA, TCC-EWMA, and TCC-DEWMA) was calculated from actual data by using equations (1,8, and 23) for different value of (λ = 0.05, 0.1, 0.2).

Table 1: Total dissolved solid data and value of (EWMA, DEWMA, TCC-EWMA, and TCC-DEWMA)

NO		EWMA			DEWMA			TCC EWMA			TCC DEWMA		
	λ	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2
	TDS	Zi	Zi	Zi	Zi"	Zi"	Zi"	TCC-Zi	TCC-Zi	TCC-Zi	D-Zi	D-Zi	D-Zi
		118.7	118.7	118.7	118.7	118.7	118.7	119.8	119.8	119.8	119.2	119.2	119.2
1	120.2	118.7	118.8	119.0	118.7	118.7	118.7	119.8	119.8	119.9	119.2	119.2	119.2
2	123.6	119.0	119.3	119.9	118.7	118.8	119.0	120.0	120.2	120.6	119.2	119.2	119.3
3	118.6	119.0	119.2	119.6	118.7	118.8	119.1	119.9	120.1	120.2	119.2	119.2	119.4
4	121.0	119.1	119.4	119.9	118.7	118.9	119.3	120.0	120.1	120.4	119.2	119.2	119.5
5	117.0	119.0	119.2	119.3	118.7	118.9	119.3	119.8	119.8	119.7	119.2	119.2	119.5
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
45	123.6	118.5	118.4	118.2	118.6	118.7	118.3	118.6	118.4	118.2	118.7	118.7	118.3
46	128.8	119.0	119.4	120.3	118.6	118.8	118.7	119.1	119.4	120.3	118.7	118.8	118.7
47	120.8	119.1	119.6	120.4	118.7	118.8	119.0	119.2	119.6	120.4	118.7	118.8	119.0
48	121.0	119.2	119.7	120.5	118.7	118.9	119.3	119.3	119.7	120.5	118.7	118.9	119.3
49	115.0	119.0	119.2	119.4	118.7	119.0	119.3	119.1	119.2	119.4	118.7	119.0	119.3
50	113.8	118.7	118.7	118.3	118.7	118.9	119.1	118.8	118.7	118.3	118.7	118.9	119.1

From table (1) it shows that the all-dissolved solid data and the values of the (EWMA, DEWMA, TCC-EWMA, and TCC-DEWMA) for different value of λ and k, where (λ = 0.05, 0.1, 0.2, 0.25) and (k=2, 3).

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Table 2: UCL and LCL of (EWMA, DEWMA, TCC EWMA, and TCC DEWMA)

λ	0.05	0.1	0.2	0.25	0.2
K	2	2	2	2	3
UCL EWMA	120.14	120.79	121.75	122.17	123.30
LCL EWMA	117.16	116.52	115.55	115.14	114.00
Range	2.98	4.27	6.20	7.03	9.30
UCL DEWMA	119.06	119.23	119.51	119.63	119.92
LCL DEWMA	118.30	118.13	117.85	117.73	117.44
Range	0.76	1.11	1.66	1.90	2.48
UCL TCC EWMA	121.3	122.28	123.76	124.39	125.76
LCL TCCEWMA	116.75	115.77	114.29	113.66	111.93
Range	4.55	6.51	9.47	10.73	13.83
UCL TCC-DEWMA	119.61	119.85	120.21	120.37	120.79
LCL TCC-DEWMA	118.49	118.25	117.89	117.73	117.31
Range	1.12	1.60	2.32	2.64	3.48

Table (2) determines the value of (UCL and LCL) of (EWMA, DEWMA, TCC-EWMA, and TCC-DEWMA) for the value of (λ = 0.05, 0.1, 0.2, 0.25) and (k=2, 3), it seen that the greater value of UCL of (TCC-EWMA) is (125.76) and LCL is (111.93) for parameters when (k=3, and λ=0.2), and the UCL is for the same parameter of DEWMA is (119.92), and LCL is (117.44). For all option of (k=2,3 and λ=0.05, 0.1, 0.2, 0.25) the small range between UCL and LCL of DEWMA control chart is (0.76, 1.11, 1.66,1.9, 2.48), it is a good control process.

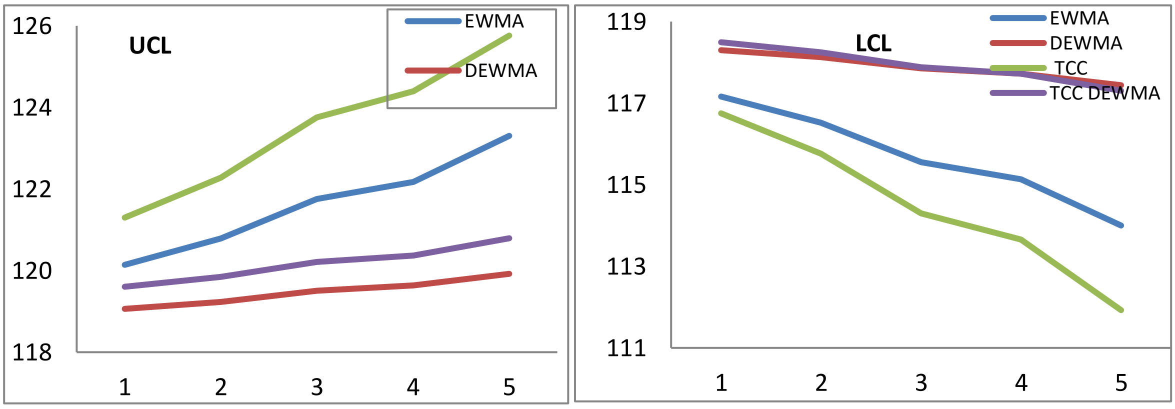

Figure 1: UCL and LCL for (EWMA, DEWMA, TCC-EWMA, and TTC-DEWMA) control chart

Figure (1) shows that the chart of different values of UCL and LCL of (EWMA, DEWMA, TCC-EWMA, and TTC-DEWMA) control chart for the different parameter values of (k=2, 3) and λ=(0.05, 0.1, 0.25, 0.5), it seen that the LCL of TCC-EWMA is the smallest value is (116.75, …,111.925), but the LCL of DEWMA is (118.30, …, 117.44), as shown in table (2) and figure (5).

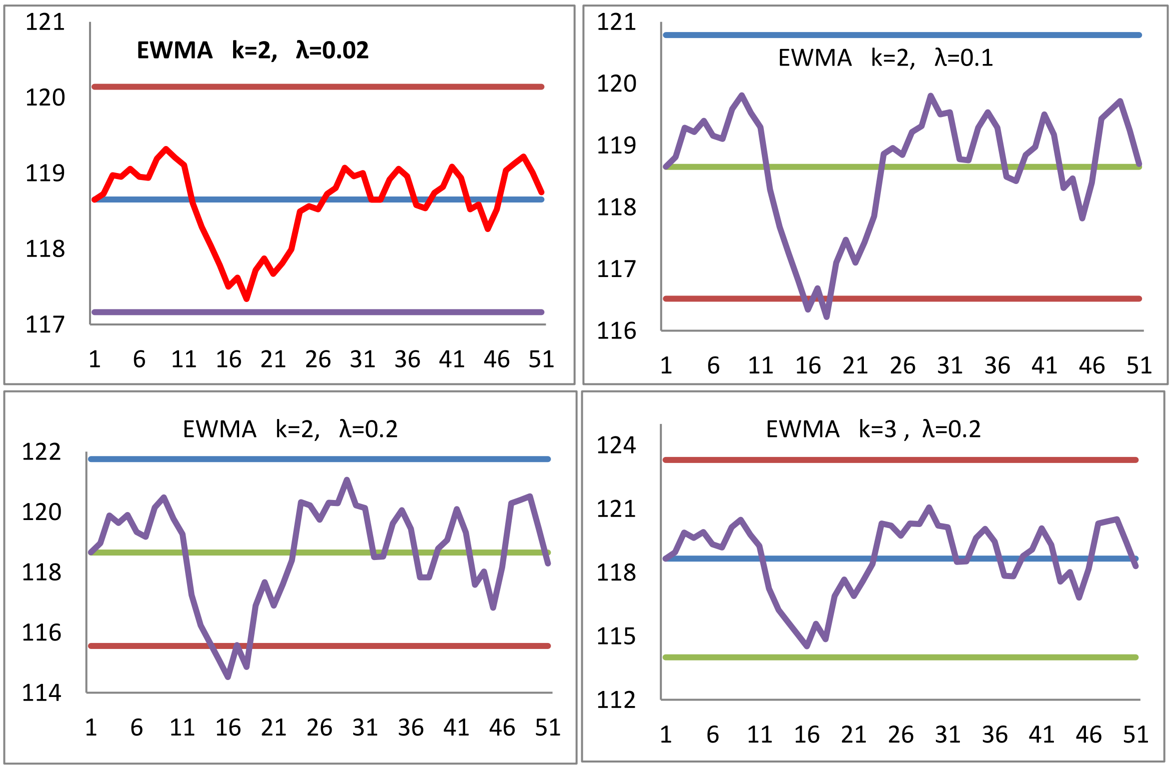

Figure 2: EWMA control chart for (k=2, 3 and λ=0.02, 0.1, 0.2)

From figure (2) show that the EWMA Control chart it seen that (2) points are out of the LCL when (k=2, λ=0.1), and if (k=2, λ=0.2), there are (3) points out of LCL.

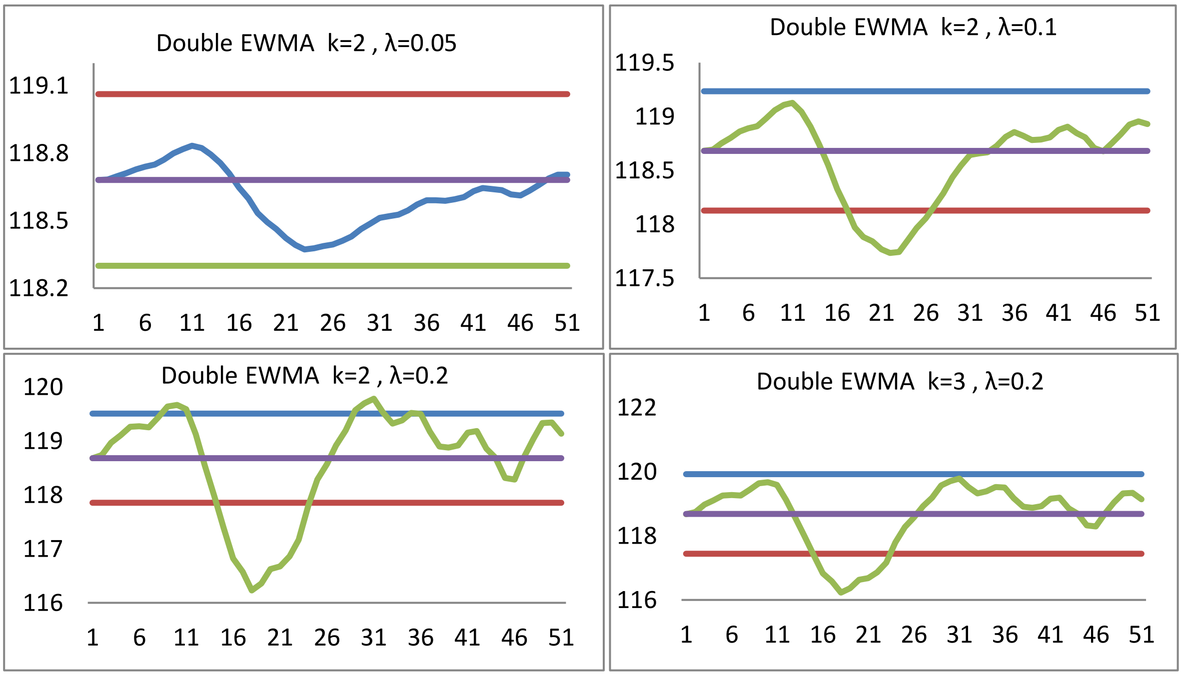

Figure 3: DEWMA control chart for (k=2, 3 and λ=0.02,0.1,0.2)

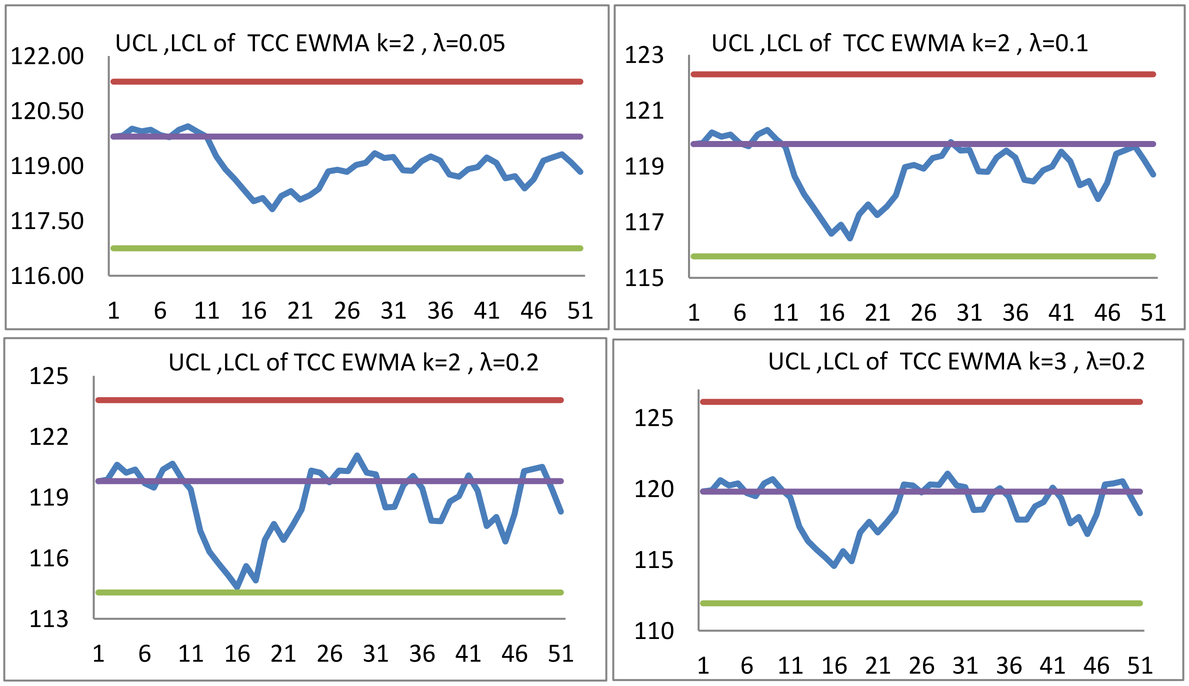

Figure 4: TCC EWMA control chart for (k=2, 3 and λ=0.02, 0.1, 0.2)

From figure (3) show that the DEWMA control chart it seen that more than (2) points are out of the UCL and LCL when (k=2,3, λ=0.1,0.2).

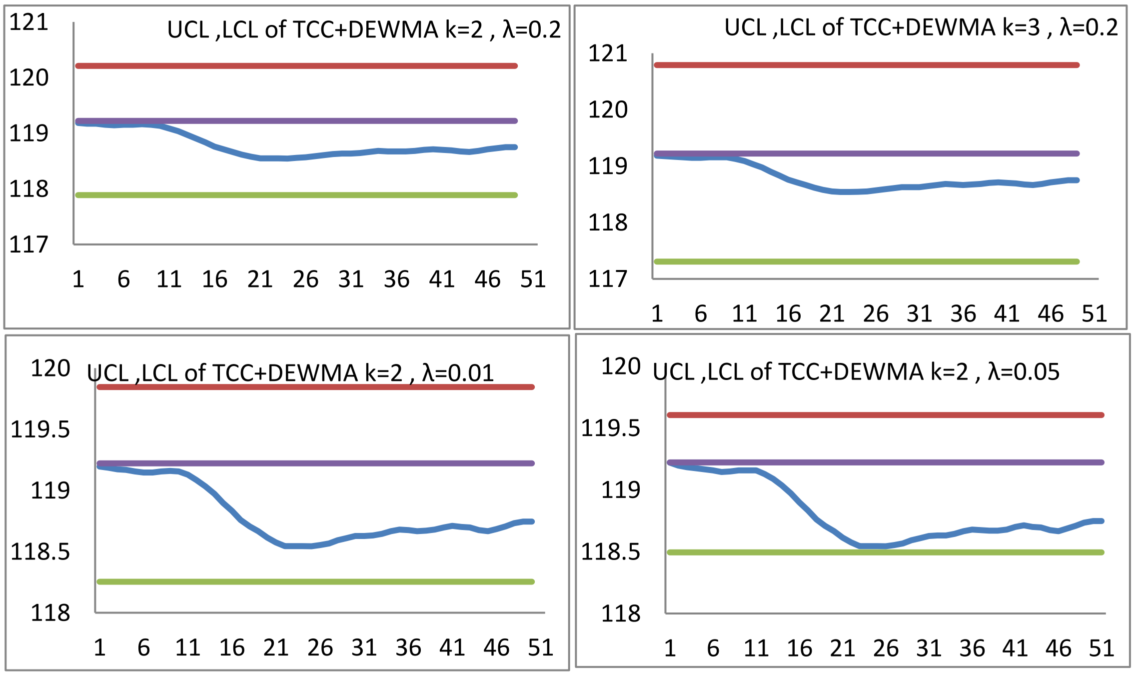

Figure 5: TCC DEWMA control chart for (k=2, 3 and λ=0.02, 0.1, 0.2

From figure (4) and (5) there are no point out of the control limit.

Table 3: Prediction value of DEWMA

#	TDS	*St*	*S′t*	*at*	*bt*	*Yt*
	120.20	118.65	118.68	118.62	-0.01	119.18
1	120.20	118.96	118.74	119.19	0.06	120.87
2	123.60	119.89	118.97	120.81	0.23	120.39
3	118.60	119.63	119.10	120.16	0.13	120.68
4	121.00	119.91	119.26	120.55	0.16	119.54
5	117.00	119.32	119.27	119.37	0.01	119.12
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
45	123.60	118.17	118.29	118.05	-0.03	117.68
46	128.80	120.30	118.69	121.90	0.40	121.87
47	120.80	120.40	119.03	121.76	0.34	122.17
48	121.00	120.52	119.33	121.71	0.30	122.05
49	115.00	119.41	119.35	119.48	0.02	119.78
50	113.80	118.29	119.13	117.45	-0.21	117.47

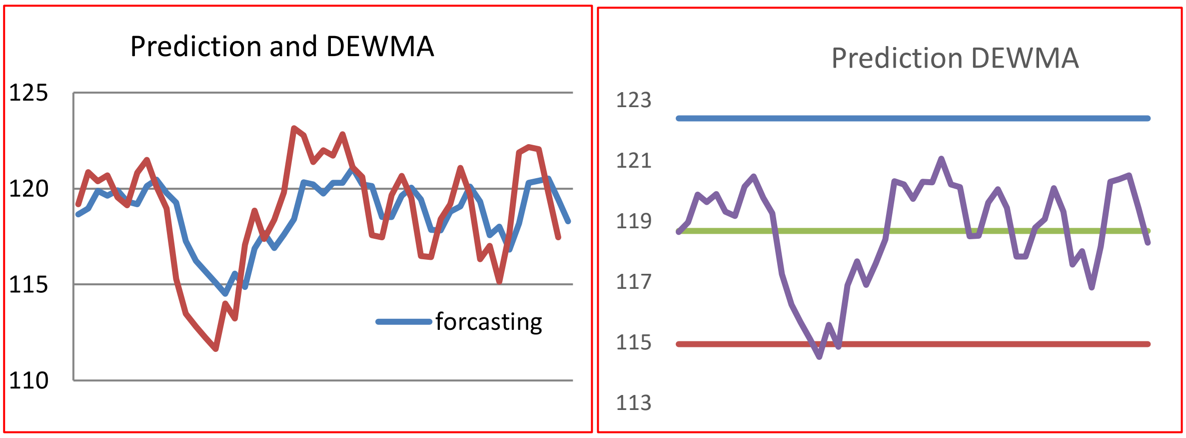

Figure 6: Prediction control chart

By using equations (18,19) determined the value prediction as shown in the table (3) and it is possible to create three control charts of DEWMA with a linear relationship, by equation (22) we calculated the UCL and LCL of the control chart for a linear prediction is value is (118.68,114.94), from figure (6) shows that two points out of LCL, the forecasting of DEWMA linear prediction can be used as an alternative of DEWMA control chart.

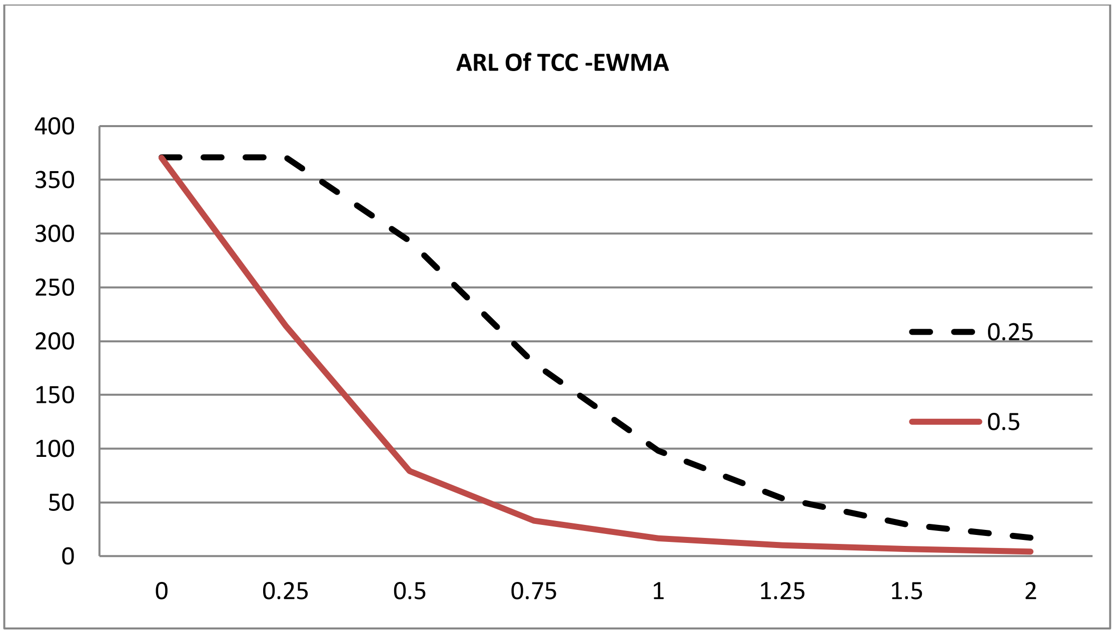

Figure 7: ARL of TCC- EWMA control chart

Table 4: ARL of TCC -EWMA

Shift	λ=0.5	λ=0.25
0	370.94	370.49
0.25	370.94	214.33
0.5	293.17	79.19
0.75	179.51	32.95
1	97.82	16.61
1.25	53.66	9.88
1.5	29.68	6.74
2	17.12	4.17

Table 5: ARL of DEWMA, where k=3

ARL DEWMA K=3					
Shift	λ=0.05	λ=0.1	λ=0.2	λ=0.25	λ=0.5
0	377.36	377.36	377.36	377.36	377.36
0.1	58.30	114.29	188.28	213.89	292.93
0.2	9.21	25.97	64.26	82.70	170.49
0.3	2.76	8.04	24.15	33.88	93.38
0.4	1.42	3.39	10.58	15.50	51.97
0.5	1.1	1.88	5.37	7.97	30.29
0.6	1	1.32	3.13	4.58	18.45
0.7	1	1.11	2.08	2.92	11.79
0.8		1.03	1.55	2.06	5.50
0.9		1.01	1.27	1.59	4.01
1			1.13	1.32	2.40
1.2			1.02	1.08	1.67
1.4			1	1.0	1.31
1.6				1.0	1.13
1.8					1.1
2					1

Table (4) shows the value of ARL of TCC-EWMA for λ= (0.25, 0.5), for the different shifts, the greater value of ARL is (370.94) for (λ=0.5), and table (5) shows the value of ARL of DEWMA where the value of k=3 and λ= (0.05, 0.1, 0.25, 0.5), as shown in figure (7), it seen that increased ARL value with increasing the λ, it shows that ARL is equal (9.21) where λ=0.05 and the shift mean of the process is equal (0.2), and if λ=0.5 then ARL =170.49 for the exact value of the shift. However, the value of ARL increases and decreases with the increase in the value of the shift means of the process, as shown in figure (8) shows the chart of ARL of

DEWMA where the value of k=3 and λ= (0.05, 0.1, 0.25, 0.5) for the different shifts, it has been seen that increase ARL value with increasing the λ.

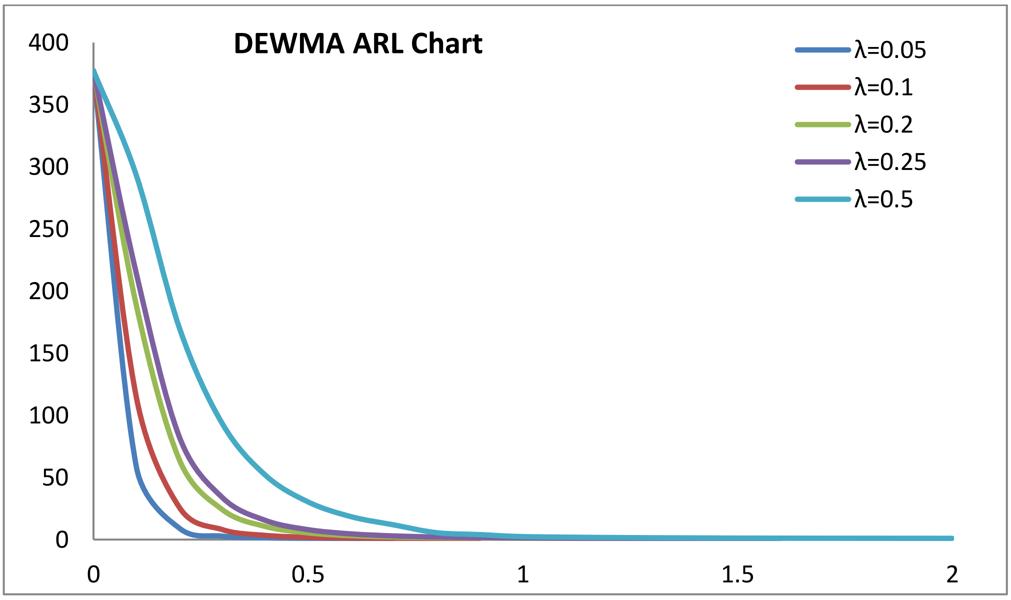

Figure 8: DEWMA ARL chart

Table 6: ARL of DEWMA where k=2,3

k	K1=3	K1=3	K1=3	K1=3	K1=3
k	K2=2	K2=2	K2=2	K2=2	K2=2
λ	λ=0.05	λ=0.1	λ=0.2	λ=0.25	λ=0.5
0	361.17	361.17	361.17	361.17	361.17
0.1	51.49	105.42	177.06	201.87	278.69
0.2	6.456	21.25	57.17	74.84	159.64
0.3	1.089	5.51	7.59	28.51	85.05
0.4	1.07	2.09	3.45	11.82	81.44
0.5	1	1.26	3.13	5.45	25.18
0.6	1	1.06	1.35	2.89	14.42
0.7	1	1.01	1.13	1.81	8.61
0.8		1	1.01	1.35	5.38
0.9		1	1.00	1.14	3.55
1		1	1	1.1	2.50
1.2			1	1	1.52
1.4				1	1.18
1.6					1.1
1.8					1
2					1

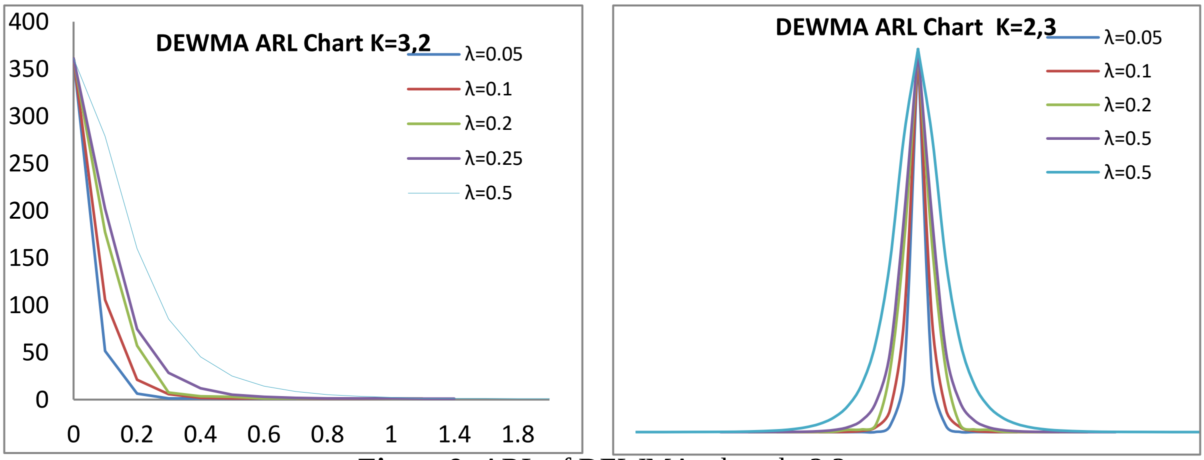

Figure 9: ARL of DEWMA where k=2,3

Table (6) shows that the value of ARL of DEWMA where the value of k=2, 3 and λ= (0.05, 0.1, 0.25, 0.5) for the different shift. It seen that the increase in the ARL value with increasing the λ, it shows that ARL is equal (6.456) where λ=0.05 and the shift mean of the process is equal (0.2), and if λ=0.5 then ARL =159.64 for the exact value of the shift. It means that the ARL decreases with the increasing shift mean of the process as shown in figure (9).

Conclusion

During a comparative study between EWMA, DEWMA, TCC-EWMA, and TCC-DEWMA, according to the principles of the Tukey's control chart and specifically created for skewed distributions, it shows that the new TCC control chart works better than the other control charts, TCC process has a significant effect on a control chart process. This study suggests an effective on EWMA, DEWMA control chart. This paper has seen that mixed TCC control with EWMA and DEWMA has a significant effect. Different length parameters, such as average run length, are used to evaluate the performance of the proposed and competing charts (ARL) According to the study, the proposed control chart consistently results in smaller. It was found that TCC control process has a significant effect EWMA and DEWMA. Further research studies may extend the scope to examine the method of efficiency comparison of the control charts, which can then be applied to the data with different distributions, DEWMA linear prediction can be used as an alternative EWMA control chart.

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