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Some Properties of Fuzzy Connected Space Defined on a Fuzzy Set

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Abstract

The primary objective of this paper is to introduce the new definitions of fuzzy separation and fuzzy connectedness in fuzzy topological space, such as (fuzzy $\tilde{s}^*\tilde{g}$- separation, fuzzy ($\tilde{s}^*\tilde{g}-\tilde{\alpha}$) separation, fuzzy $\tilde{s}^*\tilde{g}$ - connected, fuzzy ($\tilde{s}^*\tilde{g}-\tilde{\alpha}$) connected) by using the definitions of fuzzy $\tilde{s}^*\tilde{g}-$ ($\tilde{s}^*\tilde{g}-\tilde{\alpha}$) open sets and learning the interactions between them. We also explore a fuzzy hereditary and fuzzy topological properties and show that fuzzy $\tilde{s}^*\tilde{g}-$ ($\tilde{s}^*\tilde{g}-\tilde{\alpha}$)connectedness is not a fuzzy hereditary property but a fuzzy topological feature.

Keyword:- f. $\tilde{s}^*\tilde{g}$ .s (fuzzy $\tilde{s}^*\tilde{g}$ – separated) , f.($\tilde{s}^*\tilde{g}-\tilde{\alpha}$).s ,f. $\tilde{s}^*\tilde{g}$ .c(fuzzy $\tilde{s}^*\tilde{g}$ – connected) , f. ($\tilde{s}^*\tilde{g}-\tilde{\alpha}$). c , fuzzy $\tilde{s}^*\tilde{g}$ –home , fuzzy ($\tilde{s}^*\tilde{g}-\tilde{\alpha}$) home .

Introduction

The concept of fuzzy set was introduced by Zadeh[1]. And the fuzzy topological space was introduced by Chang [6].And Fuzzy connected sets in fuzzy topological spaces introduced by Chaudhuri [2].

Introduced fuzzy connected spaces defined as a fuzzy topological space $\tilde{X}$ is said to be fuzzy disconnected space if $\tilde{X}$ can be expressed as the union of two disjoint non – empty fuzzy open subsets of $\tilde{X}$. Otherwise, $\tilde{X}$ is fuzzy connected space. In this work, we recall some basic concept that we need in our work we introduce a new definition fuzzy $\tilde{s}^*\tilde{g}$- separation, fuzzy ($\tilde{s}^*\tilde{g}-\tilde{\alpha}$) separation, fuzzy $\tilde{s}^*\tilde{g}$- connected, fuzzy ($\tilde{s}^*\tilde{g}-\tilde{\alpha}$)- connected space using definitions fuzzy

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$(\tilde{s}^*\tilde{g}-\tilde{\alpha})$ open sets and study the relations among them .At last we show that fuzzy $(\tilde{s}^*\tilde{g}-\tilde{\alpha})$-connected is not – fuzzy hereditary property but fuzzy topological property. Through this paper the fuzzy topological space $(\tilde{X},\tilde{\mathrm{T}}_x)$ and $(\tilde{Y},\tilde{\mathrm{T}}_y)$ (or simply $\tilde{X}$ and $\tilde{Y}$) when $\tilde{A}$ is a fuzzy subset of $(\tilde{X},\tilde{\mathrm{T}}_x)$, int $(\tilde{A})$, $\widetilde{CL}(\tilde{A})$ which denote the interior and closure of a fuzzy set $\tilde{A}$.

1- Basic concepts of fuzzy set

Definition (1-1) [1]:- The membership function $\mu\tilde{A} : \tilde{x} \to [0, 1]$ defines a non-empty set $\tilde{X}$. and a fuzzy set $\tilde{A}$ in $\tilde{X}$. Thus we may describe this fuzzy set as.

$\tilde{A} = \{(x, \mu\tilde{A}(x): x \in X , \mu\tilde{A}(x) \leq 1\}$

I^xStands for the collection of all fuzzy sets in $\tilde{X}$. , when $I^{\tilde{x}}= \{ \tilde{A}: \tilde{A}$ is fuzzy set in $\tilde{X}$ }

Definition (1-2) [1]:- The set of all $x \in \tilde{X}$ such that a fuzzy set $\mu\tilde{A}(\tilde{x}) > 0$ and designated by the symbol $S(\tilde{A})$ is the support of a fuzzy set $\tilde{A}$.

Definition (1-3):- A fuzzy point $\tilde{P}_x^r$ in x is a unique fuzzy set whose membership function is given by $\tilde{P}_x^r(y) = \begin{cases} r , \text{if } x = y \\ 0 , \text{if } x \neq y \end{cases}$

When $0 < r \leq 1$, y is the support of $\tilde{P}_x^r(x)$.

Definition (1-4) [1]:- If $S(\tilde{A})$ is a finite set, then a fuzzy set $\tilde{A}$ is described as a finite fuzzy set.

Remark (1-5):-

1- A non-empty set $\tilde{X}$ is referred to as a crisp set since it is a fuzzy set with membership $\mu_{\tilde{x}}(x) = 1,\ \forall x \in \tilde{X}$.

2- A membership function $\mu_{\tilde{\emptyset}}(x) = 0, \forall x \in \tilde{X}$ is called an empty set and denoted by $\tilde{\emptyset}$.

Definition (1-6) [1]:- Let $\tilde{C}$ be a fuzzy set in the non-empty set $\tilde{X}$ and $\tilde{P}_x^r$ be a fuzzy point. If $\mu\tilde{P}_x^r \leq \mu\tilde{C}(x)$ for every $x \in X$ and indicated by $x \in S(\tilde{C})$, then $\tilde{P}_x^r$ is said to be in $\tilde{C}$ or that $\tilde{C}$ includes$\tilde{P}_x^r$.

Definition (1-7) [1]:- Let $\tilde{A}$ and $\tilde{B}$ by fuzzy sets of a universal set $\tilde{X}$ then

1- $\tilde{A} \subseteq \tilde{B}$ iff $\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$, $\forall x \in \tilde{X}$

2- $\tilde{A} = \tilde{B}$ iff $\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$ for all $x \in \tilde{X}$

3 - With a membership function of $\mu_{\tilde{A}^c} = 1-\tilde{A}_{\tilde{A}}(x)$, $\tilde{A}^C$ is the complement of a fuzzy set $\tilde{A}$.

4 - $\tilde{C} = \tilde{A} \cup \tilde{B}$ iff $\mu_{\tilde{C}}(x) = \max \{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}, \forall\, x \in \tilde{X}$

5 - $\tilde{D} = \tilde{A} \cap \tilde{B}$ if and only if $\mu_{\tilde{D}}(x) = \min \{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}, \forall\, x \in \tilde{X}$

6 - More specifically, for a family of fuzzy sets { $\tilde{A}_{\alpha}$: $\alpha \in \wedge$ where $\wedge$ is the any index set }

The union $\tilde{C} = \bigcup_{\alpha\in\wedge} A\alpha$ and the intersection $\tilde{D} = \bigcap_{\alpha\in\wedge} A\alpha$ and defined respectively by

$\mu_{\tilde{C}}(x) = \sup_{\alpha\in\wedge} \{\mu_{\tilde{A}}(x);\, x \in \tilde{X} \}$

$\mu_{\tilde{D}}(x) = \inf_{\alpha\in\wedge} \{\mu_{\tilde{A}}(x);\, x \in \tilde{X} \}$

Definition (1-8):- A fuzzy subset $\tilde{A}$ of fuzzy space $\tilde{X}$ is said to be

1- fuzzy $\tilde{s}^*\tilde{g}$ - closed set if $\mu\overline{\tilde{A}}(x) \le \mu\tilde{u}(x)$ wher $\tilde{A} \le \tilde{u}$ and $\mu\tilde{u}(x) \le \mu(\overline{\text{int}\,(\tilde{u})(x)})$, the set of all f. $\tilde{s}^*\tilde{g}$ .c. subsets in $\tilde{X}$ is signified by $\tilde{S}^*\tilde{G}$ C($\tilde{X}$).

The complement of an f. $\tilde{s}^*\tilde{g}$ .c (fuzzy $\tilde{s}^*\tilde{g} -$ closed) is said to be f. $\tilde{s}^*\tilde{g}$.o.s , the collection of all fuzzy $\tilde{s}^*\tilde{g}$ - open subsets in $\tilde{X}$ is designated by the symbol $\tilde{S}^*\tilde{G}$ O($\tilde{X}$).

2- The fuzzy $\tilde{s}^*\tilde{g}$ - closure of $\tilde{A}$ represents by $\tilde{s}^*\tilde{g}$ –($\overline{\tilde{A}}$) is the intersection of all f. $\tilde{s}^*\tilde{g}$. c. subset of $\tilde{X}$ which contains $\tilde{A}$.

3- A f. $\tilde{s}^*\tilde{g} - \tilde{\alpha}$.o.s if $\mu\tilde{A}(x) \le \mu$ int($\tilde{s}^*\tilde{g} - \overline{\tilde{A}}(x)$), the complement of an fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha} -$ open set is defined to be f. $\tilde{s}^*\tilde{g} - \tilde{\alpha}$. c , the family of all f. $\tilde{s}^*\tilde{g} - \tilde{\alpha}$.o. subsets of $\tilde{X}$ is denoted by $\tilde{\mathrm{T}}^{\tilde{s}^*\tilde{g}-\tilde{\alpha}}$. The intersection of all fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha} -$ closed sets containing $\tilde{A}$ is represented by the symbol $\widetilde{cl}_{\tilde{s}^*\tilde{g}-\tilde{\alpha}}(\tilde{A})$.

Definition (1-9):- A function f: $\mu\tilde{X} \to \mu\tilde{y}$ allegedly is fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) continuous iff the fuzzy inverse image of each f.o.s (fuzzy open set) of $\tilde{Y}$ is a f. $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$). o. subset of $\tilde{X}$.

2- Fuzzy connectedness in a fuzzy topological space.

We introduce the concept of fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) - connected space and study some of their properties. Also we study that fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) - connected is not hereditary property but fuzzy topological property.

Definition (2-1):- Uncertain topological space if two disjoint f. $\tilde{s}^*\tilde{g}$ .o. subsets $\tilde{E}$ and $\tilde{F}$ of $\tilde{X}$ exist, then $\tilde{X}$ is a fuzzy $\tilde{s}^*\tilde{g}$ – separation space. However, min { $\mu\tilde{E}(x)$, $\mu\tilde{s}^*\tilde{g}$ –$\overline{\tilde{F}}(x)$}= $\tilde{\emptyset}$ and min { $\mu\tilde{F}(x)$,$\mu\tilde{s}^*\tilde{g}$ – $\widetilde{cl}(\tilde{E})(x)$} = $\tilde{\emptyset}$

Definition (2-2):- A fuzzy topological space $\tilde{X}$ is a fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha} -$ separation space iff there exists two disjoint fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha} -$ open subsets $\tilde{F}$ and $\tilde{E}$ of $\tilde{X}$, whenever min { $\mu\tilde{E}(x)$, $\mu\widetilde{cl}\tilde{s}^*\tilde{g} - \tilde{\alpha}\,(\tilde{F})(x)$} = $\widetilde{\emptyset}$ and min { $\mu\tilde{F}(x)$, $\mu\widetilde{cl}\tilde{s}^*\tilde{g} - \tilde{\alpha}\,(\tilde{E})(x)$ } = $\widetilde{\emptyset}$.

Remark (2-3):-

1-Every fuzzy open set is an $\tilde{s}^*\tilde{g}$ - ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – open set but the converse is not true.

Also a fuzzy separation space is fuzzy $\tilde{s}^*\tilde{g}$ –($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – separation space.

But the converse is not true

2- Fuzzy $\tilde{s}^*\tilde{g}$ – open sets and f. $\tilde{s}^*\tilde{g} - \tilde{\alpha}$ -open set are in general independent, so we'll get that Each fuzzy $\tilde{s}^*\tilde{g}$ – separation and fuzzy ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – separation space are in general independent.

Remark (2-4):-

1 - Every two disjoint a fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) - open subsets of any space, then they are fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – separation.

2 – Every two disjoint a fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – closed subset of any space, then they are fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – separation.

Because (let $\tilde{E}$ and $\tilde{F}$ are disjoint fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – closed subset of $\tilde{X}$, We already

min { $\mu\tilde{E}(x)$, $\mu\widetilde{cl}_{\tilde{s}^*\tilde{g}}$ ($\tilde{F}$)(x) } = min { $\mu\tilde{E}(x), \mu\tilde{F}(x)$ } = $\widetilde{\emptyset}$ and min { $\mu\tilde{F}(x)$, $\mu\widetilde{cl}_{\tilde{s}^*\tilde{g}}$ ($\tilde{E}$)(x)} = min { $\mu\tilde{F}(x), \mu\tilde{E}(x)$ } = $\widetilde{\emptyset}$

$\tilde{A} = \widetilde{cl}\,(\tilde{A})$ iff $\tilde{A}$ is fuzzy closed

By definition we get that $\tilde{E}$ and $\tilde{F}$ are fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – separation.

Definition (2-5):- Topological space that is fuzzy If $\tilde{X}$ cannot be described as a disjoint union of two non-empty fuzzy $\tilde{s}^*\tilde{g}$ – open sets, then $\tilde{X}$ is said to be fuzzy $\tilde{s}^*\tilde{g}$ – connected.

(i-**e** there are two fuzzy $\tilde{s}^*\tilde{g}$ that are open subsets of $\tilde{X}$, $\tilde{E}$ and $\tilde{F}$, provided that min {$\mu\tilde{F}(x), \mu\tilde{E}(x)$} = $\widetilde{\emptyset}$, max {$\mu\tilde{F}(x), \mu\tilde{E}(x)$} $\neq \tilde{X}$.

If a fuzzy topological space $\tilde{X}$ does not attain fuzzy $\tilde{s}^*\tilde{g}$ – connected space, it is alleged to be fuzzy $\tilde{s}^*\tilde{g}$ – disconnected space.

Definition (2-6):- An undefined topological space Fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha}$ – connected is a description of $\tilde{X}$ if it cannot be described as a disjoint union of two non-empty fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha}$ – open sets.

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(i-**e** there exists two fuzzy $\tilde{s}^*\tilde{g}$-$\tilde{\alpha}$ – open subsets $\tilde{E}$ and $\tilde{F}$ of $\tilde{X}$ provided that min $\{\mu\tilde{F}(x), \mu\tilde{E}(x)\} = \tilde{\emptyset}$, max $\{ \mu\tilde{F}(x), \mu\tilde{E}(x)\} \neq \tilde{X}$.

Uncertain topological space If $\tilde{X}$ does not reach fuzzy $\tilde{s}^*\tilde{g}$ – $\tilde{\alpha}$ – connected space, then it is fuzzy $\tilde{s}^*\tilde{g}$ – $\tilde{\alpha}$ – disconnected space ".

Remark (2-7):- 1- "Every fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha}$ – open set is $\tilde{\alpha}$ – f.o.s"

so every fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha}$ – connected space is $\tilde{\alpha}$ – f.connected space .

2 – For each fuzzy $\tilde{s}^*\tilde{g} -$ connected and fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha}$ – connected space are in general independent. As in the example (2-8)

3 – A fuzzy connected space is fuzzy $\tilde{s}^*\tilde{g}$ – $(\tilde{s}^*\tilde{g} - \tilde{\alpha})$ – connectedness space.

4 – Unspecified subset it is claimed that $\tilde{A}$ of $\tilde{X}$ is a fuzzy $\tilde{s}^*\tilde{g}$ – $(\tilde{s}^*\tilde{g} - \tilde{\alpha})$ disconnected set if it is the union of two non-empty fuzzy $\tilde{s}^*\tilde{g}$ – $(\tilde{s}^*\tilde{g} - \tilde{\alpha})$ segregated sets with the symbol $\tilde{s}^*\tilde{g}$ – $(\tilde{s}^*\tilde{g} - \tilde{\alpha})$ in them. This leads to the statement that $\tilde{A}$ is fuzzy $\tilde{s}^*\tilde{g}$ – $(\tilde{s}^*\tilde{g} - \tilde{\alpha})$ connected If it isn't fuzzy or unconnected, it's $\tilde{s}^*\tilde{g}$ – $(\tilde{s}^*\tilde{g} - \tilde{\alpha})$.

Example (2-8):-

1 – let $\tilde{X}$ = { (6, 0.3) , (7 ,0.3) , (8 , 0.3) , (9 , 0.3) }

On $\tilde{T}$ = { $\tilde{X}$, $\tilde{\emptyset}$, { (6, 0.3) , (7, 0.3) , (8,0.3) , (9, 0.0)} , { (6, 0.3) , (7, 0.3) , (8 , 0.0) , (9, 0.0)} . Then $\tilde{X}$ is fuzzy $\tilde{s}^*\tilde{g}$ – connected space (because there exists two fuzzy $\tilde{s}^*\tilde{g}$ – open subsets $\tilde{F}$ and $\tilde{E}$ of $\tilde{X}$ such that $\tilde{F}$ = { (6 , 0.3), (7, 0.0), (8 , 0.0) ,(9, 0.0)}

And $\tilde{E}$ = { (6 , 0.0) , (7, 0.3),(8,0.0),(9, 0.0)} whenever min $\{ \mu\tilde{F}(x), \mu\tilde{E}(x)\} = \tilde{\emptyset}$ and max $\{ \mu\tilde{F}(x) , \mu\tilde{E}(x)\} \neq \tilde{X}$, but not fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha}$ – connected .

2 – let $\tilde{X}$ = { (1, 0.5) , (2 , 0.5) , (3, 0.5) , (4 , 0.5) }

On $\tilde{T}$ = { $\tilde{X}$, $\tilde{\emptyset}$, { (1, 0.5) , (2 , 0.0) , (3 ,0.0) , (4 , 0.0)}

Hence $\tilde{X}$ is fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha}$ – connected and $\tilde{X}$ is fuzzy $\tilde{\alpha}$ – connected space, fuzzy $\tilde{s}^*\tilde{g} -$ unconnected space.

Theorem (2-9):- A fuzzy subset $\tilde{E}$ of $\tilde{X}$ is fuzzy $\tilde{s}^*\tilde{g}$ – $(\tilde{s}^*\tilde{g} - \tilde{\alpha})$ – disconnected if it is defined as a union of two non-empty fuzzy $\tilde{s}^*\tilde{g}$ – $(\tilde{s}^*\tilde{g} - \tilde{\alpha})$ – separated subsets of.

Proof :- $\Longrightarrow$ suppose that $\tilde{E}$ is fuzzy $\tilde{s}^*\tilde{g}$ – disconnected , then $\mu\tilde{E}$ = max $\{ \mu\tilde{A} , \mu\tilde{B} \}$ where $\tilde{A}$ and $\tilde{B}$ are two fuzzy $\tilde{s}^*\tilde{g}$ – disjoint non – empty closed sets ,

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Assume that $\widetilde{A}$ and $\widetilde{B}$ are fuzzy $\tilde{s}^*\tilde{g}$ – separated subsets of $\tilde{X}$.

Min $\{\mu\widetilde{A}, \mu\tilde{s}^*\tilde{g} - \widetilde{cl}\,(\widetilde{B})\} = \min\{\min\{\mu\widetilde{A}, \mu\widetilde{E}\}, \mu\tilde{s}^*\tilde{g} - \widetilde{cl}\,(\widetilde{B})\} =$

$\min\{\mu\widetilde{A}(x), \mu\tilde{s}^*\tilde{g} - \widetilde{cl}_{\tilde{\tau}^*_{\widetilde{E}}}(\widetilde{B})(x)\} = \min\{\mu\widetilde{A}(x), \mu\widetilde{B}(x)\} = \widetilde{\emptyset}$,

so $\min\{\mu\widetilde{B}(x), \mu\tilde{s}^*\tilde{g} - \widetilde{cl}_{\tilde{\tau}_{\widetilde{E}}}(\widetilde{A})(x)\} = \min\{\mu\widetilde{B}(x), \mu\widetilde{A}(x)\} = \widetilde{\emptyset}$

$\Leftarrow$ suppose that $\mu\widetilde{E} = \max\{\mu\widetilde{A}(x), \mu\widetilde{B}(x)\}$ where $\widetilde{A}$ and $\widetilde{B}$ are fuzzy $\tilde{s}^*\tilde{g}$ – open sets disjoint non – empty fuzzy $\tilde{s}^*\tilde{g}$ – separated subsets of $\tilde{X}$.

we have $\min\{\mu\widetilde{A}(x), \mu\tilde{s}^*\tilde{g} - \overline{\widetilde{B}}(x)\} = \min\{\min\{\mu\widetilde{A}(x), \mu\widetilde{E}(x)\}, \mu\tilde{s}^*\tilde{g} - \overline{\widetilde{B}}(x)\} = \widetilde{\emptyset}$ and so that $\min\{\mu\widetilde{B}(x), \mu\tilde{s}^*\tilde{g} - \widetilde{cl}_{\tilde{\tau}_{\widetilde{E}}}(\widetilde{A})(x)\} = \min\{\min\{\mu\widetilde{B}(x), \mu\widetilde{E}(x)\}, \mu\tilde{s}^*\tilde{g} - \overline{\widetilde{A}}(x)\} = \widetilde{\emptyset}$,

we get that $\widetilde{E}$ is the union of non – empty fuzzy $\tilde{s}^*\tilde{g}$ – separated subsets of $\widetilde{E}$,Thus $\widetilde{E}$ is fuzzy $\tilde{s}^*\tilde{g}$ – disconnected. In the same way we demonstrate for fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha} -$ open set.

Corollary (2-10):- If a fuzzy space $\tilde{X}$ is fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – separation space, then $\tilde{X}$ is the union of two disjoint non – empty fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – closed subsets of $\tilde{X}$.

Proof: - let $\tilde{X} = \max\{\mu\widetilde{E}(x), \mu\widetilde{F}(x)\}$ where as $\widetilde{E}$ and $\widetilde{F}$ are fuzzy $\tilde{s}^*\tilde{g}$ – separated sets, then fuzzy $\tilde{s}^*\tilde{g} - \overline{\widetilde{E}} = \min\{\mu\tilde{s}^*\tilde{g} - \widetilde{cl}\,(\widetilde{E})(x), \max\{\mu\widetilde{E}(x), \mu\widetilde{F}(x)\}\} = \max\{\min\{\mu\tilde{s}^*\tilde{g} - \widetilde{cl}\,(\widetilde{E})(x), \mu\widetilde{E}(x)\}, \min\{\mu\tilde{s}^*\tilde{g} - \widetilde{cl}\,(\widetilde{E})(x), \mu\widetilde{F}(x)\}\} = \min\{\mu\tilde{s}^*\tilde{g} - \widetilde{cl}\,(\widetilde{E})(x), \mu\widetilde{E}(x)\} = \widetilde{E}$ (by def . 1-2) so $\widetilde{E}$ is fuzzy $\tilde{s}^*\tilde{g}$ – closed set.

Similarly $\widetilde{F}$ is fuzzy $\tilde{s}^*\tilde{g}$ – closed set.

We demonstrate the same style for the fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha} -$ open set.

As above noted hence that $\tilde{\alpha}$ –fuzzy connected is fuzzy topological property.

Corollary (2-11) :- A fuzzy space $\tilde{X}$ is a union of two disjoint non – empty fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – open subsets of $\tilde{X}$, then $\tilde{X}$ is fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – disconnected.

Proof: - suppose that $\tilde{X} = \max\{\mu\widetilde{E}(x), \mu\widetilde{F}(x)\}$ where as $\widetilde{E}$ and $\widetilde{F}$ are disjoint non – empty fuzzy $\tilde{s}^*\tilde{g}$ – open sets , then $\widetilde{E} = \widetilde{F}^C$ is fuzzy $\tilde{s}^*\tilde{g}$ – closed . So $\tilde{X}$ is fuzzy $\tilde{s}^*\tilde{g}$ – disconnected.

If P is any property in $\tilde{X}$. Then we call P hereditary if it appears in a relative fuzzy topological space if we say P is not – hereditary.

Remark (2-12):- It is not a genetic trait for the fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – connectivity.

Similar to the example:

Example (2-13) :- 1) Let $\tilde{X}$ = { (r ,0.12),(s , 0.12) , (t ,0.12) ,(u , 0.12) }

And $\tilde{\tau}$ = {$\tilde{\emptyset}$, $\tilde{X}$, {(r ,0.12) ,(s, 0.0) ,(t,0.0) ,(u,0.0)}, {(r , 0.0),(s, 0.12) ,(t ,0.12),(u,0.12)} , { (r ,0.12) ,(s,0.0) ,(t,0.4),(u,0.12)} ,{(r,0.0) ,(s,0.0) ,(t,0.12) ,(u,0.12)}

Then $\tilde{X}$ is fuzzy $\tilde{s}^*\tilde{g}$ – connected space because ∃ { (r ,0.12) ,(s ,0.0) ,(t ,0.0) ,(u ,0.12)} , {(r,0.0) ,(s,0.0),(t ,0.12) ,(u,0.0)}

are fuzzy $\tilde{s}^*\tilde{g}$ – open sets such that min { (r ,0.0) ,(s,0.0) ,(t,0.12) ,(u,0.0)} ,{(r ,0.0) , (s,0.0) , (t ,0.12) ,(u ,0.0) } = $\tilde{\emptyset}$ and max {(r ,0.12) ,(s,0.0) ,(t,0.12) ,(u ,0.0) },{(r ,0.12) ,(s ,0.0) ,(t ,0.0) ,(u,0.0) } ≠ $\tilde{X}$.

If $\tilde{A}$ = { (r, 0.12), (s ,0.12) } ≤ $\tilde{X}$ and $\tilde{\tau}_{\tilde{A}}$ = {$\tilde{\emptyset}$, $\tilde{A}$, {(r ,0.12) , (s,0.0) } , {(r,0.0) ,(s ,0.12)}}

Then ($\tilde{A}$, $\tilde{\tau}_{\tilde{A}}$) is not fuzzy $\tilde{s}^*\tilde{g}$ – connected ∃ {(r,0.12) ,(s,0.0) } ,{(r,0.0) ,(s ,0.12) } are fuzzy $\tilde{s}^*\tilde{g}$ – open sets whenever min { (r ,0.12) ,(s,0.0) }, {(r ,0.0) ,(s,0.12) } = $\tilde{\emptyset}$ and max { (r,0.12) ,(s,0.0) } ,{ (r ,0.0) ,(s ,0.12) } = $\tilde{X}$

2) let $\tilde{X}$ = { (k ,0.9) ,(l ,0.9),(m ,0.9) ,(n ,0.9) } on $\tilde{\tau}$ = { ($\tilde{\emptyset}$, $\tilde{X}$, {(k ,0.0) ,(l ,0.9) ,(m ,0.0) , (n ,0.0) } ,{(k ,0.0) ,(l ,0.9) ,(m ,0.9) ,(n ,0.0) } ,{(k ,0.9) ,(l ,0.9) ,(m ,0.0) ,(n ,0.0)} , {(k ,0.0). (l,0.9) ,(m ,0.9) ,(n ,0.0)} ,{(k ,0.0) ,(l ,0.0) ,(m,0.9) ,(n ,0.9) } , {(k ,0.9) ,(l ,0.9) ,(m,0.9) , (n ,0.0)} ,{(k ,0.0) ,(l ,0.9) ,(m ,0.9) ,(n ,0.9)}} ,

Then $\tilde{X}$ is fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha}$ – connected space, but if $\tilde{A}$ = {(l ,0.9) ,(m ,0.9) } ≤ $\tilde{X}$ and $\tilde{\tau}_{\tilde{A}}$ = {$\tilde{\emptyset}$, $\tilde{A}$,{ (l ,0.9) ,(m ,0.0) } ,{(l ,0.0) ,(m ,0.9)}} so $\tilde{A}$ is not fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha}$ – connected space because ∃ { (l ,0.9) ,(m ,0.0)} ,{(l ,0.0) ,(m ,0.9)} are fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha}$ – open sets ,

Min { (l,0.9) ,(m ,0.0) },{ (l ,0.0) ,(m ,0.0) } = $\tilde{\emptyset}$ max { (l,0.9) ,(m ,0.0) },{(l ,0.0) ,(m,0.9)} = {(l ,0.9) ,(m ,0.9)}

Definition (2-14):- A map $\tilde{f}$: ($\tilde{X}$, $\tilde{\tau}_{\tilde{X}}$) → ($\tilde{Y}$, $\tilde{\tau}_{\tilde{y}}$) allegedly is fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – homeomorphism (fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – home . for short) if

(1) $\tilde{f}$ is bijective map .

(2) $\tilde{f}$ and $\tilde{f}^{-1}$ are fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – continuous .

Let P be any property in ($\tilde{X}$, $\tilde{\tau}_{\tilde{X}}$) if P is carried by fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – home . to another space ($\tilde{Y}$, $\tilde{\tau}_{\tilde{y}}$) we say P is fuzzy topological property.

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Now, we introduce the main result about a fuzzy topological property the fuzzy $\tilde{s}^*\tilde{g}$ – ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – connected.

Theorem (2-15):- A fuzzy $\tilde{s}^*\tilde{g}$ – connected space is a fuzzy topological property.

Proof: - A $\tilde{f} : (\tilde{X} , \tilde{T}_{\tilde{x}}) \rightarrow (\tilde{Y} , \tilde{T}_{\tilde{y}})$ be fuzzy $\tilde{s}^*\tilde{g}$ – home and space $\tilde{X}$ is fuzzy $\tilde{s}^*\tilde{g}$ – connected space.

To demonstrate this, we must $(\tilde{Y} , \tilde{T}_{\tilde{y}})$ be fuzzy $\tilde{s}^*\tilde{g}$ – connected space.

If $(\tilde{Y} , \tilde{T}_{\tilde{y}})$ be fuzzy $\tilde{s}^*\tilde{g}$ – disconnected space , then there exists two disjoint non – empty fuzzy $\tilde{s}^*\tilde{g}$ – open subsets of $\tilde{Y}$, $\tilde{E}$ and $\tilde{F}$ are fuzzy subsets of $\tilde{Y}$ such that min { $\mu\tilde{E}(x)$, $\mu\tilde{s}^*\tilde{g} - \bar{\tilde{F}}(x)$} = $\tilde{\emptyset}$ = min {{ $\mu\tilde{F}(x)$, $\mu\tilde{s}^*\tilde{g} - \tilde{cl}\ (\tilde{E})(x)$ } = $\tilde{\emptyset}$ and $\tilde{E} \neq \tilde{\emptyset}$, $\tilde{F} \neq \tilde{\emptyset}$; as $\tilde{f}$ is fuzzy $\tilde{s}^*\tilde{g}$ – continuous , Ours has $\tilde{f}^{-1}(\tilde{E}) = \tilde{E}_1$ and $\tilde{f}^{-1}(\tilde{F}) = \tilde{F}_1$ where $\tilde{E}_1$ and $\tilde{F}_1$ are fuzzy $\tilde{s}^*\tilde{g}$ – open in $\tilde{X}$.

Min {$\mu\tilde{E}_1(x)$, $\mu\tilde{s}^*\tilde{g} - \overline{\tilde{F}_1}(x)$} = $\tilde{\emptyset}$, min { $\mu\tilde{F}_1$, $\mu\tilde{s}^*\tilde{g} - \overline{\tilde{E}_1}(x)$}= $\tilde{\emptyset}$

Hence $\tilde{X}$ is fuzzy $\tilde{s}^*\tilde{g}$ – disconnected but that is contradiction

Since max {$\mu\tilde{F}_1(x)$, $\mu\tilde{E}_1(x)$} = max { $\mu\tilde{f}^{-1}(\tilde{F}_1)(x)$, $\mu\tilde{f}^{-1}(\tilde{E}_1)(x)$} = $\tilde{f}^{-1}$ (max {$\mu\tilde{F}_1(x)$, $\mu\tilde{E}_1(x)$})

Hence $\tilde{X}$ is fuzzy $\tilde{s}^*\tilde{g}$ – disconnected, $\tilde{f}^{-1}(\tilde{Y}) = \tilde{X}$, we get the assume is not true .

Then $(\tilde{Y} , \tilde{T}_{\tilde{y}})$ is fuzzy $\tilde{s}^*\tilde{g}$ – connected space .

Theorem (2-16):- A fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha} -$ connected space is a fuzzy topological property.

Proof: - A $\tilde{f} : (\tilde{X} , \tilde{T}_{\tilde{x}}) \rightarrow (\tilde{Y} , \tilde{T}_{\tilde{y}})$ be fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha} -$ home and space $\tilde{X}$ is fuzzy ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) connected space . So, we must demonstrate that $(\tilde{Y} , \tilde{T}_{\tilde{y}})$ be fuzzy ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) connected space

If $(\tilde{Y} , \tilde{T}_{\tilde{y}})$ be fuzzy ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) disconnected space , then there exists two disjoint non – empty fuzzy ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) – open subsets of $\tilde{Y}$, $\tilde{E}$ and $\tilde{F}$ are subsets of $\tilde{Y}$ such that min { $\mu\tilde{E}(x)$, $\mu\tilde{cl}_{\tilde{s}^*\tilde{g}-\tilde{\alpha}}(\tilde{F})(x)$ } =$\tilde{\emptyset}$ = min {$\mu\tilde{F}(x)$, $\mu\tilde{cl}_{\tilde{s}^*\tilde{g}-\tilde{\alpha}}(\tilde{E})(x)$} and $\tilde{E} \neq \tilde{\emptyset}$, $\tilde{F} \neq \tilde{\emptyset}$, as , $\tilde{F}$ is fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha}$ – continuous We already have $\tilde{f}^{-1}(\tilde{E}) = \tilde{E}_1$ and $\tilde{f}^{-1}(\tilde{F}) = \tilde{F}_1$ where $\tilde{E}_1$ and $\tilde{F}_1$ are fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha}$ – open in $\tilde{X}$. min { $\mu\tilde{E}_1(x)$, $\mu\tilde{cl}_{\tilde{s}^*\tilde{g}-\tilde{\alpha}}(\tilde{F}_1)(x)$} = $\tilde{\emptyset}$, min { $\mu\tilde{F}_1(x)$, $\mu\tilde{cl}_{\tilde{s}^*\tilde{g}-\tilde{\alpha}}(\tilde{E}_1)(x)$ } = $\tilde{\emptyset}$

Hence $\tilde{X}$ is fuzzy ($\tilde{s}^*\tilde{g} - \tilde{\alpha}$) disconnected but conditions since max { $\mu\tilde{F}_1(x)$, $\mu\tilde{E}_1(x)$ } = max {$\mu\tilde{f}^{-1}(\tilde{F}_1)(x)$, $\mu\tilde{f}^{-1}(\tilde{E}_1)(x)$} = $\tilde{f}^{-1}$ (max {$\mu\tilde{F}_1(x)$, $\mu\tilde{E}_1(x)$})

Hence $\tilde{X}$ is fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha}$ – disconnected , $\tilde{f}^{-1}(\tilde{Y}) = \tilde{X}$, We get that the assumption is not true .Then ($\tilde{Y}$, $\tilde{T}_{\tilde{y}}$) is fuzzy $\tilde{s}^*\tilde{g} - \tilde{\alpha}$ – connected space.

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