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On-iαw – Closed-Sets and -iαw – Continuous Functions

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Abstract

The main purpose of this article is to explain the notation of iαw- closed sets and iαw- open sets in topological space. Many properties and relationships among those sets are investigated. We also discovered that there is no relationship between iαw- closed sets and α- closed, i- closed, iα- closed sets. Additionally, the notions such as, $i\alpha w -$ continuous function , $i\alpha w -$ closed function, $i - \alpha w -$ open function, perfectly, $i\alpha w -$ continuous function , contra $i\alpha w -$ continuous function and strongly $i\alpha w -$ continuous function had been introduced. Also, we prove that each $i \propto w -$ continuous function in any topological space is $\alpha w -$ continuous function, each continuous function in any topological space is $i\alpha w -$ continuous function and there is no relationship between $\alpha -$ continuous function and $i\alpha -$ continuous function with $i\alpha w -$ continuous function as well as some of its features. Finally, every strongly $i\alpha w -$ continuous is $i\alpha w -$ continuous, every perfectly $i\alpha w -$ continuous is strongly $i\alpha w -$continuous and perfectly continuous function is contra $i\alpha w -$ continuous had been concluded.

Keywords: iαw- closed set, iw- closed set, iαw- continuous function, iw- continuous function, w- continuous function.

حول المجاميع المغلقة من النمط-iαw والدوال المستمرة من النمط-iαw

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الخلاصة

الهدف الاول من هذا البحث هو تقديم صنف المجاميع المغلقة من النمط-iαw والمجاميع المفتوحة من النمط- iαw في الفضاءات التبولوجية. عدة خصائص وعلاقات بين هذه المجاميع تم اعطائها. ايضا برهنا بانه لا توجد علاقة بين المجاميع المغلقة من النمط-iαw ، المجاميع المغلقة من النمط-i، المجاميع المغلقة من النمط-α والمجاميع المغلقة من النمط-iα ، بالاضافة الى ذلك قمنا بدراسة الدوال المستمرة من النمط-iαw ، الدوال المغلقة من النمط-iαw ، الدوال المفتوحة من النمط-iαw، الدوال المستمرة التامة من النمط-iαw والدوال المستمرة بقوة من النمط-iαw ودراسة بعض خصائصها. كذلك برهنا بان كل دالة مستمرة من النمط-iαw تكون مستمرة من النمط-αw ، وكل دالة مستمرة تكون مستمرة من النمط-iαw ، ولا توجد علاقة بين الدوال المستمرة من النمط-α والدوال المستمرة من النمط-iαw والدوال المستمرة من النمط-αw . واخيرا، برهنا بان كل دالة مستمرة بقوة من النمط-iαw تكون مستمرة من النمط-iαw ، وكل دالة مستمرة تامة من النمط-iαw تكون مستمرة من النمط-iαw .

الكلمات مفتاحية : مجموعة مغلقة من النمط-iαw ، مجموعة مغلقة من النمط-iw ، دالة مستمرة من النمط-iαw ، دالة مستمرة من النمط-iw ، دالة مستمرة من النمط-w .

Introduction

Many mathematicians have devised a variety of generalizations of closed sets in recent years. Levine [8], first defined and investigated considered the concept of generalized closed sets. Maki Devi and Balachandram [4], [10] defined generalized α-closed maps. Mohammed and Askander [2] presented the idea of i-closed sets. Sundaram and Sheik John [15] and recently M. Parimala, R. Udhayakumar and R. Jeevitha [13] studied αw-closed sets. Mohammed and Kahtab [9] defined the concept of iα- closed sets in topological spaces. In 2014, [3], Benchalli, Patil and Nalwad, introduced the concept of generalized wα-closed sets. In 2015, [14], Patil, Bechalli and Pallavi, introduced the concept of star wα-closed sets in topological spaces. In this study, a new class of closed sets is introduced, having the common adjective between the iα-closed and w-closed sets, called it iαw- closed set. This paper also defines the iαw- continuous function. Throughout this paper, $\tau, \tau^{i\alpha}$ and$\tau^{\alpha w}$, denotes the topology of open set, iα-open, and αw-open respectively on the set G. Any subset H of G. $cl(H), int(H)$, Denotes closure set of H, interior set of H, we use the lower case letter of i, $i\alpha$, αw, to the mention symbol to deal with topologyτ^{i}, $\tau^{i\alpha}$and$\tau^{\alpha w}$. Throughout this paper spaces (G,τ) and (M,σ)(or simply G and M).

Definition 1.1 A-subset H of a topological space G is referred to as an

$\alpha - open$ [11], [10] $if\ H \subseteq int(cl(int(H)))$. The complement of any $\alpha - open$ set is called $\alpha - closed\ set$.

i-open [2] if there exist U∈ $\boldsymbol{\tau}$ such that U≠ $\emptyset, \boldsymbol{G}$ and $\boldsymbol{H} \subseteq \boldsymbol{cl}(\boldsymbol{H} \cap \boldsymbol{U})$.

i α -open [9] if there exist $\boldsymbol{U} \in \boldsymbol{\tau}^{\boldsymbol{\alpha}}$ such that $\boldsymbol{U} \neq \emptyset, \boldsymbol{X}$ and $\boldsymbol{H} \subseteq \boldsymbol{cl}(\boldsymbol{H} \cap \boldsymbol{U})$.

w$\boldsymbol{-closed}$[15] $\boldsymbol{if\ cl(H) \subseteq U\ whenever\ H \subseteq U\ and\ U\ is\ semi - open\ in\ G}$. wcl(H) is the intersection of all w-closed sets which containing H.

αw- $closed$ [13]$if\ wcl(H) \subseteq U\ whenever\ H \subseteq U\ and\ U\ is\ \alpha - open\ in\ G$.

iw-$\boldsymbol{closed}$ [1] $\boldsymbol{if\ wcl(H) \subseteq U\ wheneverH \subseteq U\ and\ U\ is\ i - open\ in\ G}$.

semi-open [7] if there exist U∈ $\boldsymbol{\tau}$ such that U≠ $\emptyset, \boldsymbol{X}$ and $\boldsymbol{U} \subseteq \boldsymbol{H} \subseteq \boldsymbol{cl}(\boldsymbol{U})$. The complement of any semi-open set is called semi-closed set.

Definition 1.2. Let G and M be two topological spaces. A function $\boldsymbol{f}: \boldsymbol{G} \rightarrow \boldsymbol{M}$is known as following:

1) $w -$ continuous [15] if $f^{-1}(K)\ is\ w -$ closed set of G each closed set for K of M.

2) $\alpha w -$ continuous [13] if $f^{-1}(K)\ is\ \alpha w -$ closed set of G each closed set for K of M.

3) $iw -$ continuous [1] if $f^{-1}(K)\ is\ iw -$ closed set of G each closed set for K of M.

4) Strongly- continuous [12], [6] if $f^{-1}(K)$ is both open and closed in G each open subset is for K in M.

5) contra – continuous [5] if $f^{-1}(K)\ is$ closed set in G for every open set K in M

6) contra $w - continuous$[5], if $f^{-1}(K)\ is$ w closed set in G for every open set $K\ in\ M$

Lemma 1.3. [9]

1) $Each\ \alpha - open\ set\ is\ i\alpha - open.$

2)$Each\ semi - open\ set\ is\ i\alpha - open.$

3) $Each\ i - open\ set\ is\ i\alpha - open.$

2. $\boldsymbol{i\alpha w}$_ closed set

In this section, we introduce a new class of closed sets which is called iαw-closed set and we investigate the relationship with, closed set, w-closed set, αw-closed set and iw-closed set.

Definition 2.1. A subset H of (G,τ) is named an iαw-closed set if $wcl(H) \subseteq E$ whenever $H \subseteq E$ and E is iα- open in (G,τ). The opposite of $i\alpha w-$ closed set is named iαw- open. We refer to the universal family iαw − closed sets of a topological space G by $i\alpha wc(G)$.

Example 2.2. Suppose $G = \{1, 2, 3\}$ *and* $\tau = \{G, \emptyset, \{2\}, \{2, 3\}\}$. *Then*

$wc(G) = \{G, \emptyset, \{1\}, \{1, 3\}\}$ and $\tau^{\alpha} = \{G, \emptyset, \{2\}, \{1, 2\}, \{2, 3\}\}$. Then,

$\tau^{i\alpha} = \{G, \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$, and $i\alpha wc(G) = \{G, \emptyset, \{1\}, \{1, 3\}\}$

Example 2.3. Suppose $G = \{1, 2, 3\}$ *and* $\tau = \{G, \emptyset, \{3\}, \{1, 2\}\}$. *Then*

$wc(G) = \{G, \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$,

$\tau^{\alpha} = \{G, \emptyset, \{3\}, \{1, 2\}\}$,

$\tau^{i\alpha} = \{G, \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}\}$, and

$i\alpha wc(G) = \{G, \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$.

Theorem 2.4. Each closed-set is iαw-closed-set.

Proof. Suppose that H be an closed set in G, then, H=cl(H). Let $H \subseteq E$, E is an iα-open in G. Since "each closed set is w-closed"([13]) we get, $wcl(H) \subseteq cl(H)$ (where wcl(H) is the intersection of all w-closed sets containing H), and since G is closed, we get, $wcl(H) \subseteq cl(H) \subseteq E$. This shows that H is $i\alpha w - closed$.■

The following example shows that the converse of the aforementioned theorem need not be true.

Example 2.5. If $G = \{1, 2, 3\}$ *and* $\tau = \{G, \emptyset, \{3\}, \{1, 2\}\}$. *Then*

$C(\tau) = \{\emptyset, G, \{1, 2\}, \{3\}\}$, and

$wc(G) = i\alpha wc(G) = \{G, \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}\}$

If N={1,3}. Then N is not closed. However N is iαw-closed set.

Corollary 2.6. Any open-set is iαw-open set.

Theorem 2.7. Each iαw-closed set is αw-closed set.

Proof. Suppose that H be iαw-closed set in G and E is be any α-open set in G s.t $H \subseteq E$. By **lemma 1.3.(1),** E *be* $i\alpha - open$, $wcl(H) \subseteq cl(H) \subseteq E$. Thus, H is $\alpha w - closed$.■

The following example shows that the converse of the aforementioned theorem need not be true.

Example 2.8. If $G = \{1,2,3\} and\ \tau = \{G, \emptyset, \{2\}, \{2,3\}\} Then$

$\alpha wc(G) = \{G, \emptyset, \{1\}, \{1,3\}, \{1,2\}\}$, and

$i\alpha wc(G) = \{G, \emptyset, \{1\}, \{1,3\}\}$

If $N = \{1,2\}$, then N is αw-closed set but not iαw-closed set.

Theorem 2.9. Each an iαw-closed set is w-closed set .

Proof. Suppose that H be an iαw-closed set in G and E is be any semi − open set in G s.t $H \subseteq E$. By lemma 1.3.(2), $E\ be\ i\alpha - open$. Since H is closed, $wcl(H) \subseteq cl(H) \subseteq E$. Therefore, H is w-closed set ■

Theorem 2.10. Every an iαw-closed set is an iw-closed set

Proof. Suppose that H be an iαw-closed set in G and E be any i-open set in G s.t $H \subseteq E$. By **Lemma1.3.(3)**, E be iα-open. Since H is closed,$wcl(H) \subseteq cl(H) \subseteq E$

This demonstrates H is an $iw -$ closed set in G.■

Remark 2.11. There is no connection between α-closed [resp. iα-closed, i-closed and iαw-closed set in topological-space as shown in the following examples.

Example 2.12. Suppose $G = \{1,2,3\}\ Now,$

1) $If\ \tau = \{G, \emptyset, \{3\}, \{1,2\}\}, then\ \{2\}\ is\ an\ iaw - closed\ but\ not\ \alpha - \ closed$ $and\ not\ i\alpha - \ closed.$

2) $If\ \tau = \{G, \emptyset, \{2\}, \{2,3\}\}, then\ \{3\}\ is\ \alpha - \ closed\ and\ i\alpha - \ closed$ $but\ it\ is\ not\ iaw - closed.$

3) $If\ \tau = \{G, \emptyset, \{3\}\}, then\ \{1\}\ is\ i\ closed\ but\ it\ is\ not\ iaw - closed.$

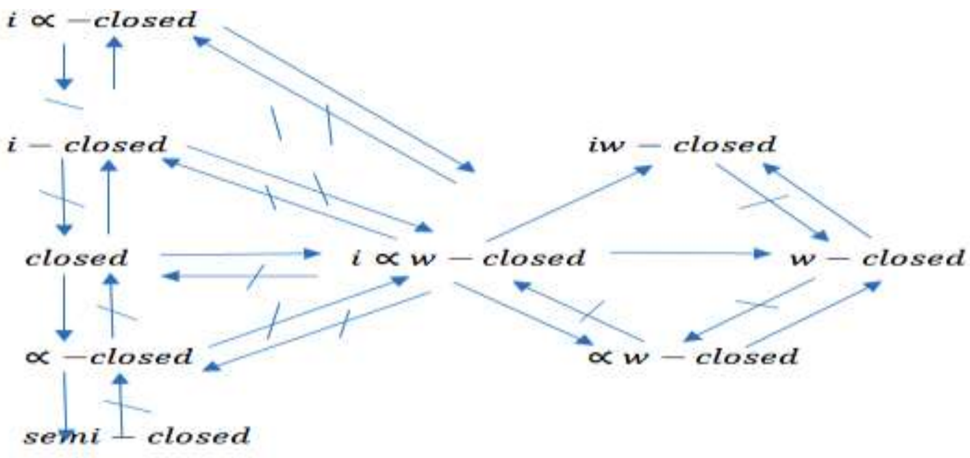

Figure 1 : The connection-between iαw-closed sets and the other classes-mentioned above

3. Properties of iαw-closed sets

We obtain several basic properties of $i\alpha w-closed$ sets.

Theorem 3.1. The intersection of two $i\alpha w-closed$ subset are $i\alpha w-closed$.

Proof. Suppose that A and B any two $i\alpha w-closed$-sets, let E be any $i\alpha-open$ in (G,τ), s.t $A\cap B\subseteq E$. Then $A\subseteq E$ and $B\subseteq E$. Since A and B are $i\alpha w-closed$ sets, $wcl(A)\subseteq E$ and $wcl(B)\subseteq E$. Therefore $wcl(A)\cap wcl(B)=wcl(A\cap B)\subseteq E$. Hence $A\cap B$ is an $i\propto w-closed$. ∎

Example 3.2. From Example (2.2) If $A=\{1\}$ and $B=\{1,3\}$, $then$ $A\cap B=\{1\}$. It is also iαw-closed set.

Theorem 3.3. If A is an iαw-closed set in G and $A\subseteq B\subseteq iawcl(A)$. Therefore, B iαw-closed set in G. But not the opposite.

Proof. Suppose A is an iαw-closed set in G. Let $B\subseteq E$ $such$ $that$ E is an $i\alpha-open$ set in G. Since $A\subseteq B$, we $have$ $A\subseteq E$. Since A is $i\alpha w-closed$ and $wcl(B)\subseteq wcl(wcl(A))=wcl(A)\subseteq E$. Therefor $wcl(B)\subseteq E$. Hence B is an $i\alpha w-closed$ in G. ∎

Example 3.4. If $G=\{1,2,3\}$ and $\tau=\{G,\emptyset,\{3\},\{1,2\}\}$. $Then$

$wc(G)=i\alpha wc(G)=\{G,\emptyset,\{1\},\{2\},\{3\},\{1,2\},\{2,3\},\{1,3\}\}$

Suppose $A=\{1\}$ and $B=\{1,3\}$, $such$ $that$ A and B are $i\alpha w-closed$ sets

But, $A\subseteq B\subset i\alpha wcl(A)$.

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Theorem 3.5. If a subset N in (G,τ) is an $i\alpha w-closed$ set then $i\alpha wcl(N)-N$ does not have any empty $i\alpha-$ $closed$ sets in (G,τ).

Proof. Suppose N is $i\alpha w-closed$set and K be a nonempty $i\alpha-closed$ subset of $wcl(N)-N$. Then $K\subseteq wcl(N)\cap(G-N)$. Since $(G-N)$ is $i\alpha-open$ and N is $i\alpha w-closed$ set. $wcl(N)\subseteq(G-N)$ therefore $K\subseteq(G-wcl(N))$.

Thus $K\subseteq wcl(N)\cap\left(G-wcl(N)\right)=\emptyset$. This implies that K= $\emptyset$. Thus $wcl(N)-N$ does not contain any nonempty $i\alpha w-closed$. ■

Theorem 3.6. Let H is $i\alpha-open$ and $i\alpha w-closed\ set, then$ H is $w-closed$.

Proof. Since $H\subseteq H\ and\ H\ is\ i\alpha-open$ and $i\alpha w-closed$, we have $wcl(H)\subseteq H$. Thus $wcl(H)=H$. Hence H is $w-closed$-set in G. ■

Theorem 3.7. Let H be an $i\alpha w-closed$ set in G. Then H is $w-closed$ iff $wcl(H)-H=\emptyset$. Which it is $i\alpha-closed$.

Proof. Suppose $H\ is\ w-closed$. Then $wcl(H)=H$ and so $wcl(H)-H=\emptyset$, which is $i\alpha-closed$. Conversely $wcl(H)-H$ is $i\alpha-closed$. Then $wcl(H)-H=\emptyset$, since H is an $i\alpha w-closed$-set-in G. That is $wcl(H)-H$ or H is $w-closed$.

4. $i\alpha w-$ Function Continuous

In this paragraph, we present the notation of an $i\alpha\ w-continuous$ function, a strongly $i\alpha w-$ continuous function and a perfectly $i\alpha w-$ continuous function.

Definition 4.1. A function $f:G\to M$ is-named $i\alpha\ w-continuous$ if f^{-1}(V) is an $i\alpha w-closed$ set of (G,τ) Regarding each closed set V of (M,σ).

Example 4.2. For $G=M=\{1,2,3\}$. Consider $\tau=\{G,\emptyset,\{3\},\{1,2\}\}, and$

$\sigma=\{M,\emptyset,\{3\},\{1,3\}\}$.

Suppose $f:(G,\tau)\to(M,\sigma)$, as the following $f(1)=1, f(2)=3, f(3)=2$. Then $f\ is\ i\alpha w-$ continuous .

Theorem 4.3. $Every\ continuous\ function$ is $i\alpha w-continuous$, but not conversely

Proof. Suppose K be closed set of (M, σ). Given that f is continuous, f^{-1}(K) is enclosed (G, τ) From Theorem (2.4), f^{-1} (K) is $i\alpha w - closed$ in (G, τ). This shows that f is iαw − continuous. ■

Example 4.4. For $G = M = \{1, 2, 3\}$. Consider, $\tau = \{G, \emptyset, \{3\}, \{1, 2\}\},\ and$
$\sigma = \{M, \emptyset, \{2\}, \{2, 3\}\}$.
Let $f: (G, \tau) \to (M, \sigma)$ be defined by $f(1) = 1, f(2) = 2, f(3) = 3$. Then
$V = \{2, 3\}\ in\ (M, \sigma)$. $f^{-1}(\{2, 3\}) = \{2, 3\}$.
Thus, f is $i\alpha w -$ continuous but not continuous function.

Theorem 4.5. Any $i\alpha w - continuous\ function\ is\ \alpha w - continuous$, but not conversely.

Proof. Suppose K closed set of (M, σ). Since f^{-1}(K) is $i\alpha w - closed$ in (G, τ). By Theorem (2.7), f^{-1}(K) is an $\alpha w - closed$ set in (G, τ). Thus, $f\ is\ \alpha w -$ continuous . ■

Example 4.6. For $G = M = \{1, 2, 3\}$. Consider,
$\tau = \{G, \emptyset, \{2\}, \{2, 3\}\}\ and\ \sigma = \{M, \emptyset, \{1, 2\}\}$.
Let $f: (G, \tau) \to (M, \sigma)$ be defined by $f(1) = 1, f(2) = 2, f(3) = 3$. Then
$V = \{1, 2\}\ in\ (M, \sigma)$. $f^{-1}(\{1, 2\}) = \{1, 2\}$.
Therfore, $f\ is\ \alpha w -$ continuous but not $i\alpha w -$ continuous.

Theorem 4.7. Any $i\alpha w - continuous\ function$ is $w - continuous$.

Proof. Suppose-K-closed-set of (M, σ). Since $f^{-1}(K)$ is an $i\alpha w - closed$ in (G, τ). By **Theorem (2.9),** $f^{-1}(K)$ is a $w -$ closed set in (G, τ). Thus $f\ is\ w -$ continuous. ■

Theorem 4.8. Every $i\alpha w - continuous\ function$ is $iw - continuous$.

Proof. Suppose K is closed set of (M, σ). Since $f^{-1}(K)$ is an $i\alpha w - closed$ set in (G, τ). By **Theorem (2.10)**, $f^{-1}(K)$ is an $iw -$ closed set in (G, τ). Thus $f\ is\ iw -$ continuous. ■

Definition 4.9. A function $f: G \to M$ is named $i\alpha w -$ closed if for each closed set F of (G, τ), $f(F)$ is an $i\alpha w - closed$-set in (M, σ).

Theorem 4.10. Every closed function is an $i\alpha w -$ closed function, but not the opposite

Proof. Every closed set being an $i\alpha w - closed$. ■

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Example 4.11. For $G = M = \{1,2,3\}$. Consider,

$\tau = \{G, \emptyset, \{2\}, \{1,2\}\}$ *and* $\sigma = \{M, \emptyset, \{3\}, \{1,2\}\}$.

Let $f: (G,\tau) \to (M,\sigma)$ be-define the-identity map, *then* f *is an* $i\alpha w -$ *closed function but* f *is not a closed* -function, because $\{1,3\}$ is closed in (G,τ) but $f(\{1,3\}) = \{1,3\}$ *is not closed in* (M,σ).

Theorem 4.12. *If a function* $f: G \to M$ *is continuouse and* $i\alpha w -$ *closed* and H which is $i\alpha w -$ *closed* of G. Then $f(H)$ is $i\alpha w -$ *closed* in M

Proof. Suppose $f(H) \subset E$ where E is it closed in M, Since f is continuouse and $f^{-1}(E)$ is it closed containing H. Thus, $cl_{i\propto}(H) \subset f^{-1}(M)$ as H is $i\alpha w -$ *closed* , since f is an $i\alpha w -$ *closed function* , $f(cl_{i\propto}(H)) \subset E$ is $i \propto w -$ *closed* , E is an closed set which implies $cl_{i\propto}\left(f\left(cl_{i\propto}(H)\right)\right) \subset E$ and hence $cl_{i\propto}\left(f\left((H)\right)\right) \subset E$ so $f(H)$ is $i\alpha w -$ closed in M. ■

Definition 4.13. A function $f: G \to M$ is known as an $i\alpha w -$ *open* function if for each open set E of (G,τ), $f(E)$ is an $i\alpha w -$ *open* set in (M,σ).

Theorem 4.14. *For any bijection* $f: G \to M$ the statements can be written as follows:

$f^{-1}: M \to G$ is $i\alpha w -$ *continuous*.

f is an $i\alpha w -$ *open* -function.

f is an $i\alpha w -$ *closed* $-$ *function* .

Proof. $(1) \to (2)$, *Assumse H an* open set in G . Since f^{-1} *is* $i\alpha w -$ continuous, then $(f^{-1})^{-1}(H) = f(H)$ is $i\alpha w -$ *open* in M and so f is $i\alpha w -$ *open*.

$(2) \to (3)$, let F a closed set in any G, then $G\backslash F$ opened in G. Since f is $i\alpha w -$ open, $f(G\backslash F)$ is $i\alpha w -$ *open* in M. But $f(G\backslash F) = M\backslash f(F)$ is $i\alpha w -$ *open* in M. Therefore $f(F)$ is $i\alpha w -$ *closed* implies that f is $i\alpha w -$ *closed*-function.

$(3) \to (1)$, . *Let* F a closed set in any G. By assumption $f(F)$ is an $i\alpha w -$ *closed* in M but $f(F) = (f^{-1})^{-1}(F)$. Therefore f^{-1} is continuous.

Definition 4.15. A function $f: G \to M$ is named strongly $i\alpha w -$ continuous if each situation were its opposite, $i\alpha w -$ open set in M opened in G.

Theorem 4.16. Every strongly $i\alpha w$ −continuous it is continuous, but not conversely.

Proof. Suppose f is strongly $i\alpha w - continuous$ and N open-set in M. By Corollary (2.6), then N be $i\alpha w - open$ in M. Since f is strongly $- i\alpha w -$ continuous , $f^{-1}(N)$ is open in G therefore f is continuous. ∎

Example 4.17. For $G = M = \{1, 2, 3\}$ Consider,

$\tau = \{G, \emptyset, \{1\}, \{3\}, \{1, 3\}, \{1, 2\}\}$ and $\sigma = \{M, \emptyset, \{3\}, \{1, 2\}\}$.

Let $f: (G, \tau) \rightarrow (M, \sigma)$ be define by $f(1) = 1, f(2) = 2, f(3) = 3$. Then f is continuous but not $strongly - i\alpha w - continuous$, because $\{2, 3\}$ $is\ i\alpha w - open\ in\ M$ but $f^{-1}(\{2, 3\}) = \{2, 3\}$ is not open in G.

Corollary 4.18. Every $strongly - i\alpha w - continuous$ is $i\alpha w - continuous$.

Theorem 4.19. Every $strongly - continuous$ is $strongly - i\alpha w - continuous$ but not conversely.

Proof. Let f be strongly continuous and U is $i\alpha w -$ open in M, since f is $i\alpha w -$ continuous, f^{-1}(U) opened-in G. Thus f is strongly $- i\alpha w -$ continuous .

Example 4.20. For $G = M = \{1, 2, 3\}$ Consider,

$\tau = \{G, \emptyset, \{3\}, \{2, 3\}\}$ and $\sigma = \{M, \emptyset, \{3\}\}$.

Let $f: (G, \tau) \rightarrow (M, \sigma)$ be defined the identity map, then f is $strongly - i\alpha w - continuous$ but not $strongly - continuous$. The subset $\{3\}$ $is\ open\ in\ M$, $f^{-1}(\{3\}) = \{3\}$ is open in G and it is not closed in G.

Definition 4.21. A function $f: G \rightarrow M$ is perfectly $i\alpha w -$ continuous if each situation were its opposite, $i\alpha w - open$-set in M is both open and closed in G.

Theorem 4.22. Every perfectly $i\alpha w -$continuous is- $strongly\ i\alpha w -$ continuous.

Proof. Assume that f is $perfectly\ i\alpha w - continuous$, let N be any $i\alpha w - open$ set in M. Suppose f is $perfectly\ i\alpha w - continuous$, f^{-1} (N) opened in G. Therefore f is $strongly\ i\alpha w - continuous$. ∎

The following example shows that the converse of the aforementioned theorem need not be true.

Example 4.23. For $G = M = \{1, 2, 3\}$ Consider

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$\tau = \{G, \emptyset, \{3\}, \{1,3\}\}, \sigma = \{M, \emptyset, \{3\}\}$. Let $f: (G, \tau) \to (M, \sigma)$ be define identification map, then f is $strongly\ i\alpha w - continuous$ c and not $pervectly\ i\alpha w - continuous$, because $\{3\}\ is\ i\alpha w - open\ in\ M$ but $f^{-1}(\{3\}) = \{3\}$ is open in G but not closed in G.

Corollary 4.24. Every $perfectly\ i\alpha w - continuous$ is $i\alpha w - continuous$.

Remark 4.25. The following diagram is based on the aforementioned results.

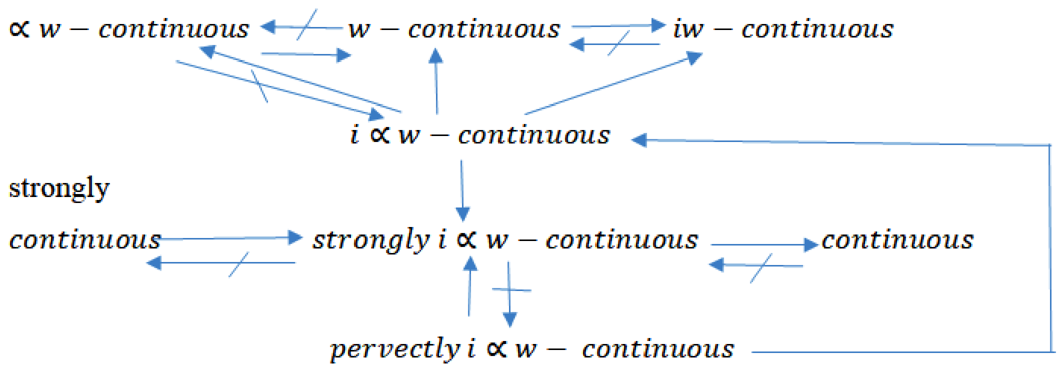

Figure 2: The connection-between iαw-continuous and the other classes-mentioned above

5. **Contra $i\alpha w$ − Continuous** Function

Definition 5.1. A function $f: G \to M$ supposed to be contra $i\alpha w -$ continuous (resp. contra $iw -$ continuous), if -each open subsets inverse-image, M is an $i\alpha w -$ closed (resp, $iw -$ closed) set in G.

Example 5.2. For $G = M = \{1,2,3\}$ $consider\ \tau = \{G, \emptyset, \{1\}\}, \sigma = \{M, \emptyset, \{3\}\}$ and

$\tau^{i\alpha} = \{G, \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$

The identity mapping is unambiguous $f: (G, \tau) \to (M, \sigma)$ is contra $i\alpha w -$ continuous and contra $iw -$ continuous.

Theorem 5.3. Any $contra -$ continuous function is- $contra\ i\alpha w - continuous$.

Proof. Let $f: G \to M$ contra continuous function And N open in M. Suppose, f is contra − continuous, then $f^{-1}(N)$ a closed set in G . By Theorem (2.4), then $f^{-1}(N)\ is\ i\alpha w -$ closed in G. Hence, f is contra $i\alpha w$ −continuous. ∎

Theorem 5.4. Every $perfectly\ continuous\ function$ is $contra\ i\alpha w - continuous$.

Proof. Let $f: G \rightarrow M$ be perfectly continuous function and V is open in M. Suppose, f is perfectly continuous, then $f^{-1}(V)$ is a cl open in G, and hence closed. Hence, it is $i\alpha w -$ closed .Thus, f is contra $i\alpha w -$ continuous .

The following example shows that contra $i\alpha w -$ continuous function need not be contra $-$ continuous and perfectly continuous

Example 5.5. For $G = M = \{1, 2, 3\}$ Consider

$\tau = \{G, \emptyset, \{3\}, \{1, 2\}\}$ and $\sigma = \{M, \emptyset, \{1\}\}$.

The identity mapping is unambiguous $f: (G, \tau) \rightarrow (M, \sigma)$iscontra $i\alpha w -$ continuous, but f is not contra $-$ continuous and not perfectly continuous . Similar results include those seen below.

Proposition 5.6. Every $contra\ i\alpha w - continuous\ function$ is$contra\ w - continuous$.

Proof. Clear by use Theorem (2.9).

Proposition 5.7. Every $contra\ i\alpha w - continuous\ function$ is$contra\ iw - continuous$.

Proof. Clear by use Theorem (2.10).

Theorem 5.8. Any $totally -$ continuous function is $contra\ i\alpha w - continuous$.

Proof. Let $f: G \rightarrow M$ be totally continuous function and V an open set in M. Since, f is totally $-$ continuous, then $f^{-1}(V)$ is a clopen set in G and hence closed . By **Theorem (2.4)**, then $f^{-1}(V)$ $is\ i\alpha w -$ closed in G. Hence, f is contra $i\alpha w -$ continuous. ■

The converse of the above theorem need not be true by the following example.

Example 5.9. If $G = M = \{1, 2, 3\}$ Consider $\tau = \{G, \emptyset, \{3\}\}$ and $\sigma = \{M, \emptyset, \{1\}\}$. The identity mapping is unambiguous $f: (G, \tau) \rightarrow (M, \sigma)$is contra $i\alpha w -$ continuous, but f is not totally $-$ continuous because for open subset $f^{-1}\{1\} = \{1\} \notin \mathrm{CO(G)}$.

Conclusions

It is concluded in this work that:

Each $\mathrm{i\alpha w} -$ closed-set is $\mathrm{\alpha w} -$ closed, each an $\mathrm{i\alpha w} -$ closed-set is $\mathrm{w} -$ closed, every an $\mathrm{i\alpha w} -$ closed-set is an $\mathrm{iw} -$ closed set.

There is no connection between $\alpha -$ closed [resp. $\mathrm{i\alpha} -$ closed , $\mathrm{i} -$ closed] and $\mathrm{i\alpha w} -$ closed in topological-space.

Every continuous function -is iαw − continuous , any iαw -continuous function is αw − continuous , any iαw − continuous function is w − continuous , every iαw − continuous function is iw − continuous.

Every strongly iαw − continuous it is continuous , every strongly − continuous it is strongly − iαw − continuous , every perfectly iαw − continuous is- strongly iαw − continuous , every perfectly iαw -continuous it is iαw − continuous , any contra − continuous function is contra iαw − continuous and every perfectly continuous function is contra iαw − continuous .

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