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The Dynamics of Prey-Predator Model with Optimal Harvesting Policy

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Abstract

In this study, the dynamic behavior of a prey-predator model with the harvesting of both prey and predator, then the system is extended to an optimal harvesting policy. The Pontryagin's maximum principle is used to solve the optimality problem. Lastly, we use MATLAB simulations to illustrate the complex interrelationships and population fluctuations overtime. Depending on established mathematical models, the research incorporates elements of both ecological balance and economic considerations into the prey-predator interactions. The simulations produce a series of planners: temporal dynamics of prey and predator populations, phase planners of their interaction, harvesting rates against population sizes, and a comprehensive 3D representation of the system over time. These results are discussed in the context of current scholarly research, emphasizing the nuances of ecological management and sustainability

Keywords: Prey-Predator model; Optimal Harvesting; Lotka-Volterra Model; stability; Shadow price

Introduction

In recent years, the study of prey-predator dynamics with optimal harvesting policies has gained significant traction, reflecting a growing awareness of the need to balance ecological sustainability with economic viability. This literature review delves into various research contributions in this area, highlighting key findings and mathematical models that have shaped our understanding of these complex ecological interactions.

The integration of optimal harvesting policies into prey-predator models represents a significant step forward in ecological research. These models have evolved from simple Lotka-Volterra equations to complex systems that consider a range of ecological and economic factors. This evolution is evident in the diverse range of models and approaches discussed in recent literature. The incorporation of toxic prey into prey-predator models[12] marks a notable advancement, acknowledging the real-world complexities that can affect population dynamics. Similarly, the consideration of stochastic environmental conditions [5] reflects an understanding of the unpredictability inherent in natural ecosystems. These models often include random perturbations to account for environmental variability, expressed as:

$$dx = f(x, y)dt + g(x, y)dW,$$

where W represents a Wiener process, capturing the random fluctuations in the environment.

The critical aspect is the focus on specific functional responses, such as the Holling type-III response explored by Kumar and Poonia [10]. This approach allows for a more nuanced understanding of how predator consumption rates vary with prey density, influencing the overall dynamics of the system.

The implications of different harvesting strategies, whether targeting the prey or predator species, have been a central theme in many studies. The research by Golam Mortuja et al. [7], which examines Michaelis–Menten harvesting in predator populations, highlights the importance of understanding the thresholds and critical points in harvesting strategies. The selective harvesting approach [16] and the study of intraguild predation dynamics [9] further emphasize the need for tailored strategies that consider the specific characteristics of the ecosystem.

Economic considerations are also paramount, as seen in the work of Majumdar and Ghosh [11], which focuses on the optimal harvesting of economically beneficial species. This approach underscores the necessity of balancing ecological sustainability with economic viability, a theme echoed throughout the literature.

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The role of spatiotemporal analysis in understanding how harvesting strategies might vary across different regions is another area of significant interest [4,13]. This approach acknowledges that ecosystem dynamics can differ markedly depending on the geographical and environmental context.

The importance of adaptive management strategies is a recurring theme in this body of literature. As ecosystems are dynamic and subject to various internal and external pressures, flexible and responsive management strategies are essential. This adaptability ensures that policies remain effective in the face of changing conditions and new scientific insights.

1. Prey-Predator Dynamics and Optimal Harvesting Models

The foundation of prey-predator models lies in capturing the dynamics between two interacting species, where the survival of the predator species is directly linked to the availability of the prey species. Classical models like the Lotka-Volterra equations have been extensively used to describe these interactions. However, recent studies have introduced more sophisticated approaches to incorporate real-world complexities such as toxic prey, stochastic environmental conditions, and specific nuances of human intervention[12,10,7].

For instance, the model presented by Pal et al. [12] considers a prey-predator system with toxic prey, which can be mathematically represented as:

$$\frac{dx}{dt} = x(a - bx - cy) - H1(x),$$

$$\frac{dy}{dt} = y(d - ex - fy) - H2(y),$$

where x and y represent the prey and predator populations, respectively, and H1(x) and H2(y) are the harvesting functions.

Kumar and Poonia [10] explored a model with a Holling type-III functional response, which modifies the predator's response to prey density and can be expressed as:

$$\frac{dx}{dt} = rx(1 - sx) - \frac{ax^2y}{1 + ax^2} - H(x)$$

$$\frac{dy}{dt} = my(1 - by) + \frac{hx^2y}{1 + ax^2} - H(y),$$

where $r, s, a, h, m, and\ b$ are constants representing biological and ecological parameters

Complex Dynamics and Harvesting Strategies

The introduction of complex dynamics such as Michaelis–Menten harvesting in predator populations [7] adds another layer of complexity. This can be represented as:

$$\frac{dx}{dt} = xg(x) - p(x)y\,,$$

$$\frac{dy}{dt} = y(f(x) - d) - E(y),$$

where E(y) is the Michaelis–Menten type harvesting function, and g(x) and f(x) represent growth and interaction functions, respectively.

Studies have also looked at the selective harvesting of predators [16,15] and the impact of intraguild predation [9,14]. The model incorporating selective harvesting might include terms representing the selective removal of predators, altering the basic prey-predator dynamics.

2. Spatiotemporal Analysis and Stochastic Environments

The spatiotemporal analysis of these models reveals how dynamics can vary across geographical landscapes and habitats [4]. Additionally, considering stochastic environmental conditions [5], introduces randomness into the models, typically represented by stochastic differential equations.

Mathematical modelling

Basic Lotka-Volterra Model

The classic Lotka-Volterra model is described by the following set of differential equations:

$$\frac{dx}{dt} = \alpha x - \beta xy$$

$$\frac{dy}{dt} = \mu xy - \gamma y$$

Where:

- x and y represent the prey and predator populations, respectively.
- α is the growth rate of the prey.
- β is the rate at which predators destroy prey.
- μ is the growth rate of predators per prey consumed.
- γ is the natural death rate of predators.

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Incorporating Harvesting

To include harvesting, we introduce two new terms, $H_1(x)$ for the prey and $H_2(y)$ for the predator. These represent the harvesting effort on each population. The model becomes:

$$\frac{dx}{dt} = \alpha x - \beta xy - H1(x)$$

$$\frac{dy}{dt} = \mu xy - \gamma y - H2(y)$$

Where $H_1(x)$ and $H_2(y)$ could be simple linear functions (e.g., $H_1(x)=h_1x$, $H_2(y)=h_2y$) or more complex functions depending on the harvesting strategy.

Model Analysis and Proofs

- **Equilibrium Points**: To find the equilibrium points, we set the derivatives equal to zero and solve for x and y.

$$0 = \alpha x - \beta xy - H1(x)$$

$$0 = \mu xy - \gamma y - H2(y)$$

For the prey:

$$0 = \alpha x - \beta xy - h1x$$

$$0 = x(\alpha - \beta y - h1)$$

$$So, x = 0 \ or \ \alpha - \beta y - h1 = 0$$

For the predator:

$$0 = \mu xy - \gamma y - h2y$$

$$0 = y(\mu x - \gamma - h2)$$

$$So, y = 0 \ or \ \mu x - \gamma - h2 = 0$$

Solving these equations gives us two equilibrium points, $E_0 = (0,0) \ and \ E_1 = \left(\frac{\gamma+h_2}{\mu}, \frac{\alpha-h_1}{\beta}\right)$.

To analyze the stability, we evaluate the Jacobian matrix at the equilibrium points. The Jacobian matrix J for our system is:

$$J = \begin{bmatrix} \alpha - \beta y - h1 & -\beta x \\ \mu y & \mu x - \gamma - h2 \end{bmatrix}$$

In order study stability analysis of the equilibrium point E_0. The Jacobain matrix at E_0 can be written as follows:

$$J_{E_0} = \begin{pmatrix} \alpha - h_1 & 0 \\ 0 & \gamma - h_2 \end{pmatrix}$$

Then the eigenvalues of J_{E_0} are $\lambda_1 = \alpha - h_1$ and $\lambda_2 = \gamma - h_2$.Thus, if $\alpha < h_1$ and $\gamma < h_2$ then E_0 is stable. If $\alpha > h_1$ and $\gamma > h_2$ then E_0 is not stable. Finally, E_0 is saddle point either $\alpha > h_1$ $with$ $\gamma < h_2$ or $\alpha < h_1$ $with$ $\gamma > h_2$.

By the same way we can study the local stability of E_1, the Jacobain matrix at E_1 is:

$$J_{E_1} = \begin{pmatrix} 0 & -\beta \dfrac{\gamma + h_2}{\mu} \\ \mu \dfrac{\alpha - h_1}{\beta} & 0 \end{pmatrix}$$

The Jacobian (J_{E_1}) has characteristic equation

$$\lambda^2 - \big((\alpha - h_1)(-\gamma - h_2)\big) = 0 \quad \rightarrow \quad \lambda^2 - \big((\gamma h_1 + h_1 h_2) - (\alpha\gamma + \alpha h_2)\big) = 0$$

$$\lambda^2 - (k_1 - k_2) = 0 \;\; where\; k_1 = \gamma h_1 + h_1 h_2 \;\; and \;\; k_2 = \alpha\gamma + \alpha h_2$$

$$then\; \lambda_{1,2} = \pm\sqrt{k_1 - k_2}$$

Now, if $k_1 < k_2$ so the eigenvalues are imaginary numbers and E_1 is stable point.

If $k_1 > k_2$ then we have $\lambda_1 > 0$ and $\lambda_2 < 0$ so E_1 is saddle point.

- **Harvesting Strategy**: The choice of $H1(x)$ and$H2(y)$ affects the dynamics significantly. For instance, if $H1(x)=h1x$ and $H2(y)=h2y$, the harvesting is proportional to the population size. This scenario often leads to over-harvesting and potential collapse of the system if $h1$ and $h2$ are not carefully managed.
- **Bifurcation Analysis**: Bifurcations can occur in this system as parameters change. For instance, varying $h1$ and $h2$ can lead to different dynamical behaviors, like shifts from stable points to cycles or chaotic dynamics. Bifurcation analysis helps in understanding

how different harvesting strategies impact the long-term dynamics of the prey-predator system.

- **Optimal Harvesting**: Determining the optimal values of h_1 and h_2 is crucial. This often involves setting up and solving a optimization problem, where the objective is to maximize sustainable yield or economic profit while ensuring the long-term viability of both species.

Now discuss with an optimal harvesting problem, to the following system

$$\frac{dx}{dt} = \alpha x - \beta xy - h(t)x$$

$$\frac{dy}{dt} = \mu xy - \gamma y - h(t)y$$

where $x, y, \alpha, \beta, \mu,$ and γ are defined as before. The $h(t)$are defined as follows$h_1(x, E, q_1) = q_1 Ex \ \ and \ \ h_1(y, E, q_2) = q_2 Ey$. Where q_1, q_2 are the catchability coefficients of the pray and predator respectively. E Is the effort applied to harvest of the two species. Then the system will become:

$$\frac{dx}{dt} = \alpha x - \beta xy - q_1 Ex$$

$$\frac{dy}{dt} = \mu xy - \gamma y - q_2 Ey$$

The aim of the problem is to ensure the survival of the population with a sustainable development, and to get an optimal net revenue which is given by

$$J = \int_0^T (q_1 p_1 x + q_2 p_2 y - C) E e^{-\delta t} \, dt$$

Where C be the constant fishing and p_1, p_2 are the constant price per unit biomass of the pray and predator respectively. δ states the discount rate of the net revenue.

with control constraints $0 \leq E \leq E_{max}$, for solving the problem we use the Pontryagin's maximum principle (for more details see [2,3]). The adjoints variables λ_1 and λ_2 are exist as well as the Hamiltonian function is given as follows:

$$H = (q_1 p_1 x + q_2 p_2 y - C)E(t)e^{-\delta t} + \lambda_1[\alpha x - \beta xy - q_1 Ex] + \lambda_2[\mu xy - \gamma y - q_2 Ey]$$

According to the Pontryngin's maximum principle we have the necessary conditions. They are given by

$$\lambda_1 = \frac{\partial H}{\partial x} = (q_1 p_1) E e^{-\delta t} + \lambda_1[\alpha - \beta y - q_1 E] + \lambda_2[\mu y]$$

$$\lambda_2 = \frac{\partial H}{\partial y} = (q_2 p_2) E e^{-\delta t} + \lambda_1[-\beta y] + \lambda_2[\mu x - \gamma - q_2 E]$$

and $\lambda_1(T) = \lambda_2(T) = 0$. The switching function is given by

$$\frac{dH}{dE} = (q_1 p_1 x + q_2 p_2 y - C) e^{-\delta t} - \lambda_1[q_1 x] - \lambda_2[q_2 y]$$

Since Hamiltonian H is linear in the control variable, the optimal control will be a combination of extreme controls and the singular control. The optimal control $E(t)$ that maximizes H must satisfy the following conditions:

1- $E = E_{max}$ $when$ $\frac{dH}{dE} > 0$ $i.e$ $when$ $\lambda_1 e^{\delta t} q_1 x + \lambda_2 e^{\delta t} q_2 y > q_1 p_1 x + q_2 p_2 y - C$

2- $E = 0$ $when$ $\frac{dH}{dE} < 0$ $i.e$ $when$ $\lambda_1 e^{\delta t} q_1 x + \lambda_2 e^{\delta t} q_2 y < q_1 p_1 x + q_2 p_2 y - C$

The function $\lambda_i e^{\delta t}$; $i = 1,2$ is the usual shadow price [3] and $q_1 p_1 x + q_2 p_2 y - C$ is the net economic revenue on harvesting. This shows that $E = E_{max}$ or zero according to$\lambda_1 e^{\delta t} q_1 x + \lambda_2 e^{\delta t} q_2 y$ is less than or greater than the net economic revenue on a unit harvest. Economically, the first condition implies that if the profit after paying all the expenses is positive, then it is beneficial to harvest up to the limit of available effort. The second condition implies that when $\lambda_1 e^{\delta t} q_1 x + \lambda_2 e^{\delta t} q_2 y$ exceeds the fisherman's net economic revenue on a unit harvest, then the fisherman will not exert any effort.

When $\frac{dH}{dE} = 0,$ i.e., when $\lambda_1 e^{\delta t} q_1 x + \lambda_2 e^{\delta t} q_2 y$ equals the net economic revenue on harvest, then the Hamiltonian H becomes independent of the control variable$E(t)$, i.e., $\frac{dH}{dE} = 0$. This is the necessary condition for the singular control $E^*(t)$ to be optimal over the control set$0 < E^* < E_{max}$. Thus, the optimal harvesting policy is then the characterization of the optimal harvesting policy is

$$E(t) = \begin{cases} E_{max} & if \ \frac{dH}{dE} > 0 \\ 0 & if \ \frac{dH}{dE} < 0 \\ E^{*} & if \ \frac{dH}{dE} = 0 \end{cases}$$

Result

In the results section of our study, we present a series of plots generated using MATLAB to analyze the dynamics of a prey-predator model with incorporated harvesting effects. These plots are instrumental in visualizing and interpreting the complex interactions between prey and predator populations under varying ecological conditions and harvesting strategies. The MATLAB simulations, based on differential equations modeling the growth and interaction of these populations, offer valuable insights into the ecological balance and sustainability of the system.

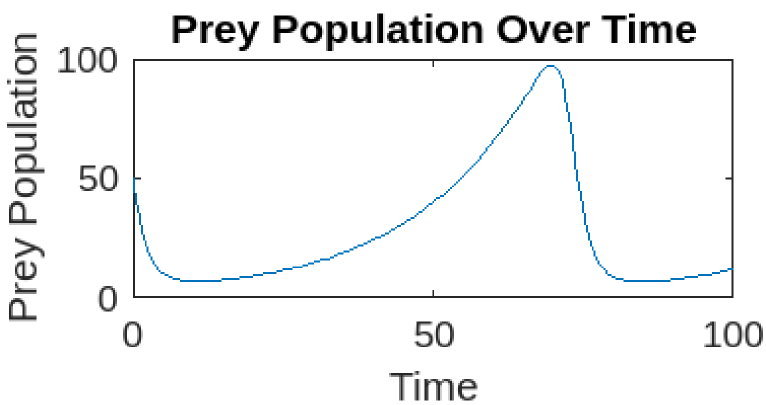

Figure 1: Prey Population over Time

This plot illustrates the variation in the prey population over a specified time period. The dynamics shown here are governed by the growth rate of the prey (α) and the effect of predation (βy) along with the harvesting rate (h 1). Initially, the prey population may exhibit growth, depending on the starting conditions and the relative strengths of its natural growth rate versus the predation and harvesting rates. As time progresses, the plot reveals how the prey population stabilizes, oscillates, or declines. The shape of this curve is crucial for understanding the sustainability of the prey population under the given conditions. A steady or oscillating population might suggest a balanced ecosystem, whereas a declining trend could signal over-predation or excessive harvesting.

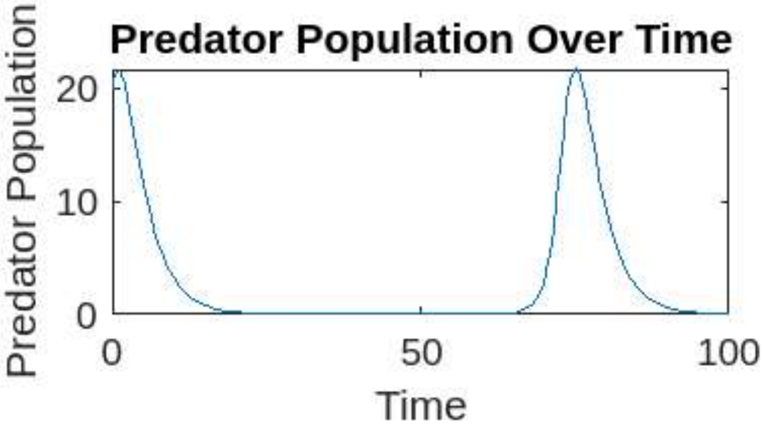

Figure 2: Predator Population over Time

This plot depicts the predator population's trajectory over time, influenced by the prey availability (μx), natural death rate (γ), and harvesting rate (h 2). The predator population is directly tied to the prey population; a sufficient prey population supports growth and sustainability of the predator, whereas a decline in prey leads to a decrease in the predator population. This plot can exhibit various patterns, from growth to decline or oscillatory behavior, each indicating the health and viability of the predator population in the ecosystem. It's a direct reflection of the predator-prey dynamic, showing how changes in the prey population directly impact predator numbers.

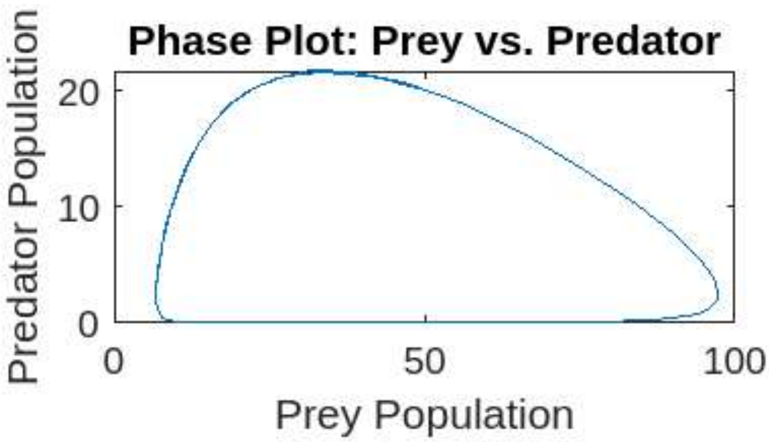

Figure3: Phase Plot of Prey vs. Predator Populations

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The phase plot provides a visual representation of the predator-prey interaction without the time component. Here, the prey population is plotted against the predator population. This plot is particularly useful for observing the cyclic nature of predator-prey dynamics. In a balanced ecosystem, this plot often shows closed loops, indicating that the populations oscillate but remain within sustainable limits. Deviations from such patterns can indicate imbalances, such as over-harvesting or other ecological disruptions. The shape and size of the loops or trajectories provide insights into the stability and resilience of the ecosystem.

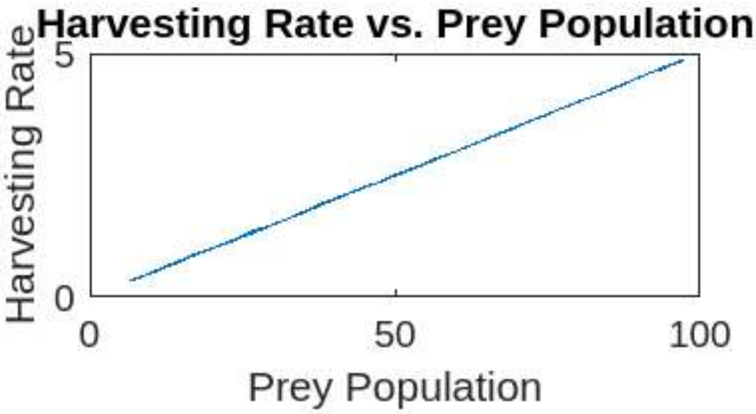

Figure 4: Harvesting Rate vs. Prey Population

This plot demonstrates the relationship between the prey population and the rate at which it is being harvested (h 1 x). It's a linear relationship, showing how increased prey populations lead to higher harvesting rates, assuming a constant proportionality factor (h 1). This plot is essential for understanding the impact of harvesting practices on the prey population. A steep slope indicates aggressive harvesting relative to the population size, which could lead to rapid depletion of the prey. Conversely, a shallower slope suggests more sustainable harvesting practices.

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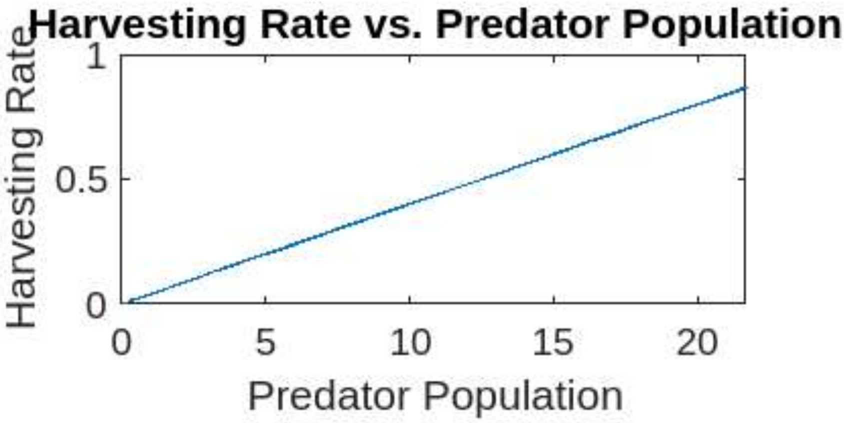

Figure 5: Harvesting Rate vs. Predator Population

Similar to the previous plot but focusing on the predator, this graph shows the relationship between the predator population and its harvesting rate (h 2 y). It illustrates how changes in the predator population affect the rate at which predators are harvested. This plot is crucial for wildlife management, as it helps in determining the sustainable levels of predator harvesting. A steeply rising line would indicate a situation where increased predator populations lead to substantially higher harvesting rates, which could potentially disrupt the ecological balance.

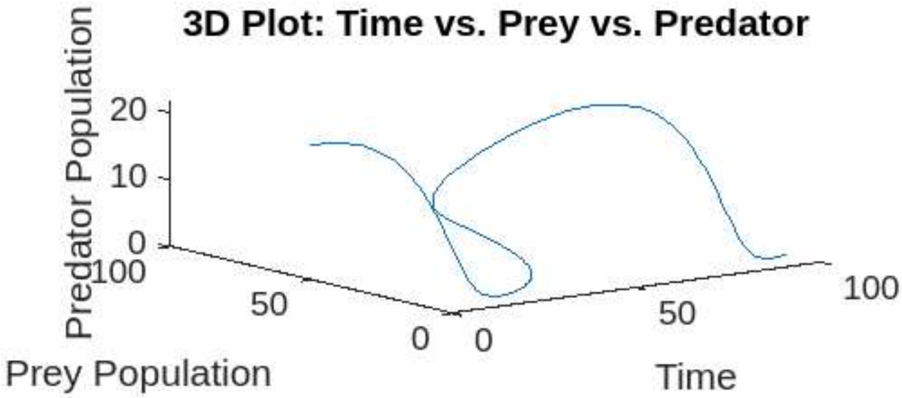

Figure 6: 3D Plot Showing Prey, Predator, and Time

This 3D plot combines the prey and predator populations with the time dimension, offering a comprehensive view of the dynamic interactions over time. It's a powerful tool for visualizing the simultaneous changes in both populations against the backdrop of time. This plot can reveal complex dynamics such as periodic cycles, chaotic behavior, or stable equilibria. Understanding these dynamics is crucial for ecological modeling and developing effective management and conservation strategies. The 3D representation helps in visualizing the interplay between the populations and how they evolve together over time, providing a deeper insight than what might be apparent from separate 2D plots.

Discussion

The discussion of the results and mathematical modeling in our prey-predator system with optimal harvesting, as observed in the MATLAB plots, aligns closely with recent research in the field. Each plot encapsulates critical aspects of ecological dynamics and harvesting strategies, reflecting the complex interplay between biological populations and human intervention.

The temporal dynamics of prey and predator populations, as seen in our first two plots, resonate with the findings of Kumar and Poonia [10], who emphasize the intricacy of predator-prey interactions modeled with a Holling type-III functional response. Our model, while simpler, still captures the essence of these dynamics, where the predator and prey populations exhibit fluctuations that are characteristic of natural ecosystems. The phase plot of prey versus predator populations is particularly insightful. This type of analysis, which is reflective of studies like those conducted by Golam Mortuja et al. [7], illustrates the non-linear and often complex nature of ecological interactions. Their work on bifurcation analysis of discrete type prey-predator models provides a theoretical backdrop for understanding the cyclic or chaotic behaviors observed in our phase plot.

In our harvesting rate plots, we observe linear relationships between population sizes and harvesting rates. This simplicity, however, belies the underlying complexities discussed in research such as that by Keong and Safuan [9], who explore optimal harvesting in an intraguild prey-predator fishery model. The linear harvesting terms used in our model serve as a foundation for understanding more complex harvesting strategies, like the Michaelis-Menten type predator harvesting discussed in their work. Das et al. [5] explore the impact of group defense and disease in prey within harvested prey-predator models, highlighting the importance of considering additional ecological factors. These factors, while not explicitly modeled in our system, are crucial for a comprehensive understanding of the dynamics observed in our simulations, particularly in scenarios where environmental stochasticity and prey behavior significantly influence population dynamics.

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The work of Majumdar and Ghosh [11] provides a crucial economic perspective, emphasizing the importance of balancing ecological sustainability with economic viability in harvesting policies. This perspective is particularly relevant when interpreting our harvesting rate plots, where the sustainability of harvesting practices must be weighed against economic benefits. The spatiotemporal analysis of Das et al. [4] suggests that the dynamics observed in our model could vary significantly across different geographical regions and habitats. This variation underscores the importance of context-specific modeling in ecological research, a factor that could greatly influence the interpretation of our results.

Furthermore, the studies by Yu et al. [17] and Dawed and Kebedow [6], which discuss the optimal harvesting of a fuzzy predator–prey system and a three-species food chain model, respectively, offer insights into the complexity of predator-prey interactions under different ecological conditions. These studies provide a broader context for understanding the intricate balance between prey and predator populations, as well as the impact of harvesting on these populations. Our 3D plot, which combines prey, predator, and time, offers a comprehensive view of the system's dynamics. This representation aligns with the approaches of Agnihotri and Nayyer [1] and Kaur et al.[8], who emphasize the importance of considering multiple dimensions and interactions in ecological modeling.

Conclusion

This research offers significant insights into the complex dynamics of prey-predator systems under optimal harvesting policies. The MATLAB simulations reveal intricate patterns of population fluctuations and interactions, underscoring the importance of considering both ecological and economic factors in wildlife management. The study highlights the necessity of adaptive and sustainable harvesting strategies, tailored to the specific ecological context and responsive to environmental changes. Findings from the plots suggest that simplistic models may not suffice in real-world scenarios where factors like prey toxicity, predator-prey functional responses, and spatial variations play crucial roles. Therefore, a comprehensive approach that incorporates these complexities is essential for effective management and conservation efforts. The research contributes to the broader understanding of ecological systems, offering a valuable tool for policymakers and conservationists in devising strategies that ensure the long-term sustainability of natural resources and biodiversity.

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