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On Bornological Structure

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Abstract

For multi criteria sequencing problem on one machine, we propose a modified branch and bound algorithm (MBAB) to find efficient (pareto optimal) solutions in this paper. The criteria are total completion time ($\sum C_j$), total lateness ($\sum L_j$), and maximum tardiness (T_{max}). A collection of n independent tasks(jobs) has to be sequenced on one machine , tasks(jobs)j (j=1,2,3,…..,n) requires processing time P_j and due data d_j . The MBAB algorithm depends on branch and bound technique. Applied examples are used to show applicability of MBAB algorithm. The MBAB algorithm is compared with complete enumeration method (CEM). Conclusions are formulated on the performance of the (MBAB) algorithm.

Keywords: Sequencing Multi-criteria, Efficient Solution, one machine, Pareto optimal Solution.

حول التراكيب البورنولوجية

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الخلاصة

في هذه المقالة ، تم تقديم نبذة تاريخية عن بعض الأعمال المتعلقة بالتركيبات الوراثية. أيضًا ، نقارن بين هيكل الطوبولوجيا وعلم الولادة. بالإضافة إلى ذلك ، تم عرض بعض الاختلافات بين علم الولادة والطوبولوجيا. أخيرًا ، تم تقديم بعض التطبيقات الحياتية لعلم الولادة ، وهذا يعني ، كيف استخدموا هذا الهيكل لحل المشكلات القائمة فيما يتعلق بالحدود والقيود.

الكلمات المفتاحية: المجموعة المقيدة، الدوال المقيدة، المجموعة شبه المقيدة، الدوال شبه المقيدة, المجاميع البرنولجيه, الزمر البرنولجية.

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Introduction

Earlier, to solve the limitation or bounded problem for any set or space, the concept of a bounded set within this set or space was given. An idea has emerged since 1971[1] to form a structure is called bornology β on a set X to solve the problem of limitation for a set X or any space in a general way. From that time if anyone want to solve the problem of boundedness for any set or structure or space they define bornology on this set or space. In other words, if we have a set X and we want to solve the problem of bounded for this set it is enough to define the collection of subsets β of X such that β covers X also, β is stable under hereditary and finite union then can solve the problem of bounded for this set in general way see[2].

Definition 1 A bornology on a set X is a family $\beta \subseteq \mathcal{P}(X)$ such that:

(i) β covers X, i.e. $X = \bigcup_{B\in\beta} B$;

(ii) β is hereditary under inclusion, i.e. if $A \subseteq B$ and $B \in \beta$ then $A \in \beta$;

(iii) β stable under a finite union, i.e. if $B_1, B_2 \in \beta$, then $B_1 \cup B_2 \in \beta$.

A pair (X, β) consisting of a set X and a bornology β on X is called a ***bornological set***, and the elements are called bounded sets. After that in 1976 [3] to limited any group they define bornology on group and study **bornological group** (BG) is a set equipped with two compatible structures of group and bornology such that the product and inverse maps are bounded. The idea of the bornological group (BG) came from starting to solve the problem of boundedness of the group. In 2012[4] they study fundamental construction for bornological group. For more information on bornological group, we can see [5,6].

Definition 2: A bornological group (G, β) is a set with two structures:

i. $(G,*)$ is a group;
ii. β is a bornology on G.

Such that the product map $\psi: (G,\beta) \times (G,\beta) \longrightarrow (G,\beta)$ is bounded and $\psi^{-1}: (G,\beta) \longrightarrow (G,\beta)$ is bounded.

In 2015 [7] they studied the further properties of bornological group. Also, every group G can be turned into a bornological group by providing it with the discrete bornology. However, the problem it's with indiscrete-bornology, there are such type of group cannot be bornological-group because the inverse map is not bounded, that means cannot solve the boundedness problem for this type of groups that means when define bornology on that groups cannot be bornological group. In 2017 [8] this problem was solved by introducing a related structure to bornological-group which is bornological semi groups, to solve the boundedness problem for that type of groups, which cannot be bornological group because the inverse map is not bounded.

Example 3 Consider the multiplicative group $(C*,\cdot)$. Let $\beta = \{Dr\,(0): r \geq 0\}$, where $Dr\,(0) = \{Z \in C*: |Z| \leq 0\}$ is the disc of radius r with the center zero. It is clear that $(C*,\beta)$ is a bornological set. But, it is not bornological group since under the inverse map, the image of a disc is not longer a disc.

Since every group is semi group, then to solve the problem of bounded for these group is define bornology on semi group.

Definition 4 A bornological semigroup (S,β) is a set with two structures:

i. $(S,*)$ is a semigroup;
ii. β is a bornology on S.

Such that the product map $\psi: (S,\beta)\times(S,\beta) \longrightarrow (S,\beta)$ is bounded.

As we know, any group can be turned over into bornological groups by providing it with the discrete-bornology. However, the problem it's with indiscrete-bornology, there are such type of group cannot be bornological-group because the product map is not bounded, that means cannot solve the boundedness problem for this type of group that means when define bornology on that group cannot be bornological group. In 2019 [9] this problem was solved by introducing a related structure to bornological-group which is semi bornological groups, to solve the boundedness problem for that type of groups which cannot be bornological group because the product map is not bounded. A ***semi bornological group*** is a group endowed with a bornology such that, for each fixed $g \in G$, the translations $l_g,\ r_g : G \to G$, are bounded, so $l_g\,(x) =$

$g.x, r_g(x) = x \cdot g$ are bounded. Also, in [10] they study further properties for semi bornological group.

Note that, when we define usual bornology or finite bornology on infinite total order set the hole set cannot belong to the collection of bornologies. So, to solve this problem the authors in [11] introduced the concept of semi bounded set.

A subset S of a bornological set (X, β) is said to be a semi bounded set if there is a bounded subset B of (X, β) such that, $B \subseteq S \subseteq \overline{B}$, where $\overline{B}$= {all upper and lower bounds of B } $\cup B$.

Note that, every bounded set is semi bounded set, but the converse is not true just in discreet bornology on infinite set. $SB(X)$ is the collection of all semi bounded subsets of (X, β).

A map f from a bornological set (X, β) into a bornological set (Y, β) is said to be

1-bounded map if the image of any bounded set of (X, β) is bounded set in (Y, β).

2-S-bounded map if the image of any bounded set in (X, β) is semi bounded set in (Y, β).

3-S*-bounded map if the image of any semi bounded set in (X, β) is bounded set in (Y, β).

Additionally, to divided the bornological set in to orbits they do action from bornological group on bornological set see [11,12,14,15]. Furthermore, to solve the problem of boundedness for soft set and fuzzy set, they define bornological fuzzy set and bornological soft set. After that, they divided bornological soft set in to orbits by study action theory of bornological soft group on bornological soft sets, the motivation for this study to make bornological soft set more practical when they put it as a classes [13].

Some Practical Applications of Bornological Structure

The most important practical application for bornology is in the spyware program KPJ. They, used to solve the problems for boundedness in specific way or with more details, for example in spyware program KPJ. Exactly, when they want to determine the person location, or the identity of the person‘s from his printed finger and his printed eye [14].

To explain this application, first. Assume that the person is the original point, to determine his signed or status. We start to study the (behaviour, vibrations, frequencies) of objects x within his domain. That means, first of all, we study the frequency of these objects x to be shower these objects within the place they define unit boll B = $\{x \in V, \|x\| < 1\}$ or unit disk B = $\{ x \in V, \|x\| \leq 1\}$. It is an absorbent disk but we need to study the behaviour for another objects. Then, we have to define another disk B_1 that is bigger (since B is ball or disk in a vector space, then it is allowed for us to multiply by a scalar in a vector space and make B bigger), so we get another absorbent disk B_1 (see figure 1). Thus by the same operation we can translate to all objects. In the end, we get collection of disks covers the place and the finite union which it is the bigger bounded set should also be within the place not outside, that means inside the collection. (A set A is absorbent disk in a vector space V if A absorbs every subset of V consisting of a single point), i.e. absorbs every subset of V.

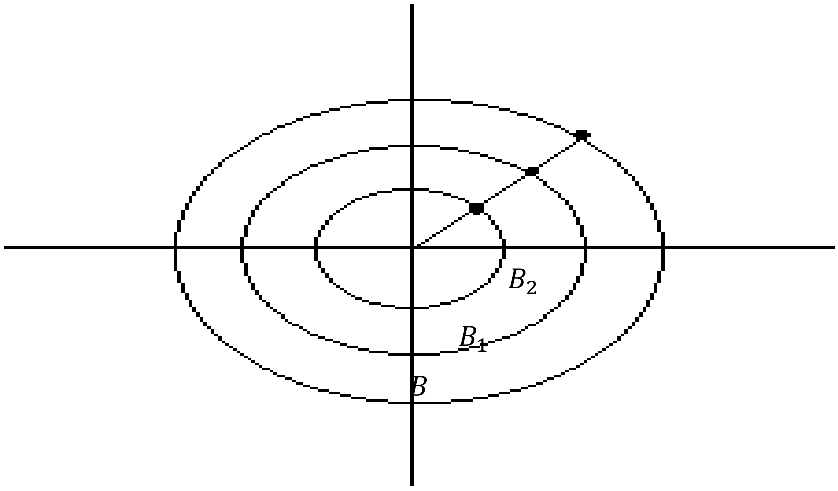

Figure 1: The behaviour of objects within his domain by introducing unit disk.

The fingerprint is one of the applications of the bornological space. Each person has his fingerprint that differs from the other. We start from the centre, around the centre there is a family of bounded sets that, the union of it, give the largest set covering the area of the thumb. Also, the hereditary property is available where a large set exists inside contains a smaller set, and so on. That means subset property is there is a transfer of all biological, formal, and life characteristics from the large set to the smaller set, and the finite union of these bounded sets gives a bounded set that belongs to the family (see Figure 2).

Figure 2: Finger print.

Another application to the iris, the pupil is the set of holes of various shapes and sizes and the distance between each other, which are located around the pupil, which acts as a (limited group) and the iris is considered one of the best ways to verify the identity of people, and no two eyes are alike in everything scientists confirmed that it is impossible for the two eyes to be exactly the same; (Figure 3).

Figure 3: Eye print [16].

There are numerous further uses for bornology, including as the provision of internet service in buildings, for more details. We consider the surface's midpoint, and since internet waves are limited intervals, a collection of these open intervals that cover the surface and are stable under finite union and hereditary property can identify the location or the structure, as shown in [2]. Also, there is a significant use for bornology in satellite broadcast systems for determining the boundaries of the broadcast area; (see Figure 5).

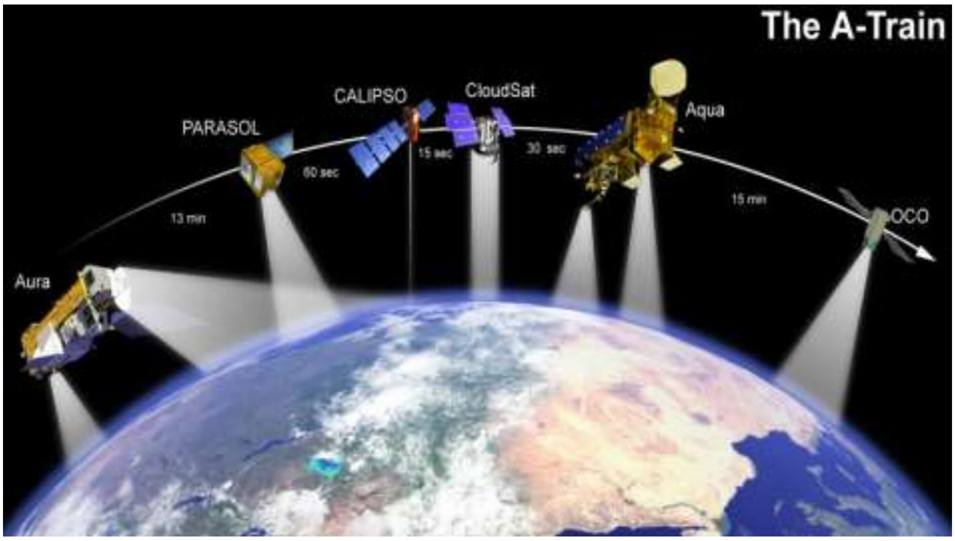

Figure 5: Satellite broadcast system.

The Comparison between bornology and topology.

In this section, we compare between bornology and topology. First of all, cannot replace the first condition(β covers X) with $X \in \beta$ (since if the hole set belong to the collection, then can cover itself) like topology ($X \in \tau$).

Because, the usual bornology is the collection of all usual bounded set(" the usual concept of bounded set for any set") on infinite set X, that $X \notin \beta$, because, the infinite set is not usual bounded set. Also, it is worth to ask. **Can replace the hereditary properties by finite intersection like topology?** No, because the finite intersection give just the smaller set. But, the hereditary properties can include all the subsets of the bounded set B and that exactly what the bornology structure working on. Furthermore, **Can replace the third condition by infinite union instead of finite union?** No,

- The element of bornology it is called bounded set. The element of topology it is called open set.
- When define bornology on a set Z, then (Z, β) is called a bornological set. When define topology on a set Z, then the pair(Z, τ) is called a topological space.
- In bornological set, we study just the fundamental construction of bornology. For example the base, sub set.

In topological space, there is study for properties. For example, compact, complete, closed set, closure set, converge of sequence and separation axiom... .

- V is a vector space. When define bornology on V is said to be bornological space. When define topology on V is said to be topological vector space.
- **In bornology,** any sub collection $\beta_\circ$ is a base if for every element of β is contain in an element of $\beta_\circ$. **In topology,** any sub collection $\tau_\circ$ is a base if for every element of $\tau_\circ$ is contain in an element of τ.
- The morphism between two bornological sets are called bounded map**. Topology.** The morphism between two topological spaces are called continuous map.
- There is no trivial (Indiscrete) bornology**. Topology** Indiscrete topology. $\tau = \{\emptyset, X\}$.
- We can define only one bornology on finite set, which it is discrete bornology. We can define many topologies on finite set.
- A map $\boldsymbol{\theta}$ between two bornological sets are isomorphism if it is one to one, onto and $\boldsymbol{\theta}, \boldsymbol{\theta}^{-1}$ are bounded**.** A map $\boldsymbol{\theta}$ between two topological space are homeomorphism if it is one to one, onto and $\boldsymbol{\theta}, \boldsymbol{\theta}^{-1}$ are continuous.
- **Bornology.** Solve the problem of bounded in sets. **Topology.** Solve the continuity of shapes.

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