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Modified Variational Iteration Method Approach for Fuzzy Initial Value Problem

Maath S. Awad [1] , Rokan Khaji [2] and Nabaa N. Hasan [3]

[1,2]Department of Mathematics, College of Science, Diyala University

[3]Department of Mathematics, College of Science, Mustansiriyah University

maath680@gmail.com

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Abstract

In this paper, an application of modified variational iteration method (MVIM) for solving homogeneous and nonhomogeneous ordinary differential equations with fuzzy initial value conditions (FDEs) are examined. This modified form is about a procedure some changes in the general formula of the variational iteration method (VIM) . MVIM showed a qualitative and excellent improvement over the standard VIM. To illustrate how this modified form is works we picked some numerical examples and solved them. The efficacy and validity of MVIM have been confirmed through results obtained.

Keywords: Modified variational iteration method, Fuzzy differential equations, Fuzzy number , Numerical solutions.

نـهج طريقة التكرار المتغيرة المعدلة لمسائل القيمة الأبتدائية الضبابية

معاذ سعدون عواد ، روكان خاجي ، نبأ نجدي حسن

قسم الرياضيات- كلية العلوم – جامعة ديالى

قسم الرياضيات- كلية العلوم – جامعةالمستنصرية

الخلاصة

في هذا البحث ، تم فحص تطبيق طريقة التكرار المتغير المعدل (MVIM) لحل المعادلات التفاضلية العادية المتجانسة وغير المتجانسة بشروط القيمة الأولية الضبابية (FDEs). هذا النموذج المعدل يتعلق بإجراء بعض التغييرات في الصيغة العامة لطريقة التكرار المتغير (VIM). نجح MVIM في إحداث تحسن نوعي وممتاز على معيار VIM. لتوضيح كيفية عمل هذا النموذج المعدل ، اخترنا بعض الأمثلة العددية وقمنا بحلها. تم تأكيد فعالية وصحة MVIM من خلال النتائج التي تم الحصول عليها.

الكلمات المفتاحية: طريقة التكرار المتغيرة المعدلة ، المعادلات التفاضلية الضبابية، العدد الضبابي ، الحلول العددية

Introduction

Surely, subjects of fuzzy differential equations (FDEs) have developed quickly in the past decades. FDEs appear in a series of scientific fields, for example medicine, particle system, population models, computational biology and control chaotic system[1]. A number of methods have been used by many researchers to find the numerical solutions of FDEs such as Euler method, Adomian decomposition, Range-kutta and Homotopy perturbation[2],[3],[4],[5].The VIM was invented by Ji-Huan He[6],[7],[8] which has high efficiency and accuracy to discovery the numerical solutions of differential equations, that it also applied by [9],[10],[11],[12].

The VIM despite all the features, it has but insight into procedures for resolving VIM shows some drawbacks, namely the frequent calculation of redundant terms, which leads to wasted effort and time. Abassy [13],[14],[15] suggested a MVIM and used it to give a series of approximate solutions for some known nonlinearities problems. The MVIM simplifies the computational work and at the same time reduce it. The proposed method can significantly and effectively improve and increase the speed of convergence. This modification introduces a very precise convergence and in some cases gives the exact solution.

This study in its entirety aims to make known to a new application of MVIM .which we can used to solve FDEs also comparison of the approximate results found by the proposed method with the exact solution that has been inserted to show the strength of the method.This paper is prepared as follows: Some important basic concepts of fuzzy set theory is illustrated in section.2,. In section.3, MVIM procedure for solving FDEs is explained. In section.4, we chose two numerical examples and resolved it to reveal and show the efficiency of the proposed modified. Finally, in the last section we drawn the conclusion.

1. Preliminaries

Some of the fundamental definitions and notations of fuzzy set theory that we needed in the work are mentioned in this section.

Definition 1.1[16]**:** A fuzzy set $\tilde{A}$ in a nonempty set of objects X with generic element x and membership function $\mu_{\tilde{A}}(x): X \to [0,1]$ can be described as a set of ordered pairs :

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X, 0 \leq \mu_{\tilde{A}}(x) \leq 1\}$$

where $\mu_{\tilde{A}}(x)$: is interpreted as degree of membership or degree of truth of x in $\tilde{A}$

Definition 1.2 [17]: A fuzzy number $\widetilde{M}$ is a mapping $\widetilde{M}: \mathbb{R} \to [0,1]$ which satisfies

(i) $\widetilde{M}$ is upper semi continuous,

(ii) $\widetilde{M}$ is a convex fuzzy set of $\mathbb{R}$ (the real line), i.e., $\mu_{\widetilde{M}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\widetilde{M}}$-$(x_1), \mu_{\widetilde{M}}(x_2)\}$, $\forall x_1, x_2 \in \mathbb{R}$, $\lambda \in [0,1]$,

(iii) $\widetilde{M}$ is normal, i.e., $height(\widetilde{M}) = 1$,

(iv) $\overline{\{s \in \mathbb{R} | \widetilde{M}(s) > 0\}}$ is compact.

Definition 1.3[18]: A fuzzy number $\widetilde{M}$ can be written in the parametric form as ordered pair $(\underline{\widetilde{M}}(s), \overline{\widetilde{M}}(s))$, $s \in [0,1]$ of functions which satisfies

(i) $\underline{\widetilde{M}}(s) \leq \overline{\widetilde{M}}(s)$, $0 \leq s \leq 1$

(ii) $\overline{\widetilde{M}}(s)$ is a bounded, decreasing and left continuous function,

(iii) $\underline{\widetilde{M}}(s)$ is a bounded, increasing and left continuous function,

Definition 1.4 [19] : If we have the following membership function of a fuzzy number $\widetilde{M}$

$$\mu_{\widetilde{M}}(x) = \begin{cases} 0, & x \leq a \\ \dfrac{x-a}{b-a}, & a \leq x \leq b \\ \dfrac{c-x}{c-b}, & b \leq x \leq c \\ 0, & c \leq x \end{cases}$$

then $\widetilde{M}$ is called triangular fuzzy number which specified by the parameters {a,b,c}, where a,c the end points and b is a peak point such as Figure 1.

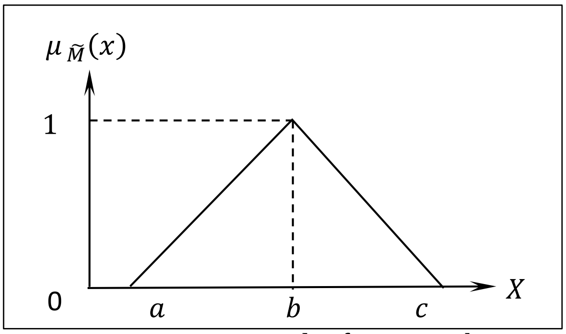

Figure 1. Triangular fuzzy number

Definition 1.5 [20] : If we have a fuzzy set $\tilde{A}$ in X then α −leval set $\tilde{A}$ is a crisp set can be written as

$$\tilde{A}_{\alpha} = \{ x \in X | \mu_{\tilde{A}}(x) \geq \alpha, \alpha \in [0,1]\}$$

Definition 1.6 [21] **:** let $\tilde{g}: \mathbb{R} \to \mathrm{E}^1$ be a function then $\tilde{g}$ is called fuzzy-valued function when E^1 represent the set of all fuzzy number ,on the other hand $\tilde{g}(s,r) = [\underline{g}(s,r), \overline{g}(s,r)]$ in the parametric format for all $s \in \mathbb{R}$ and $r \in [0,1]$.

Definition 1.7 [22] **:** Let $f(x)$ be a fuzzy function from $[a,b] \subseteq \mathbb{R}$ to $\mathbb{R}$ such that $\forall\, x \in=$ $[a,b], \tilde{f}(x)$ is a fuzzy number and $f_{\alpha}^{+}(x)$ and $f_{\alpha}^{-}(x)$ are $\alpha - level$ functions .The integral of $\tilde{f}(x)$ over $[a,b]$ is then defined as the fuzzy set.

$$\tilde{I}(a,b) = \left\{\left(\int_a^b f_{\alpha}^{-}(x)\, dx + \int_a^b f_{\alpha}^{+}(x)\, dx, \alpha\right)\right\}.$$

2. Modified Variational Iteration Method

Let us consider the next general nonlinear initial value problem to illustrate the basic idea of (MVIM), which reads as

$$L[u(t)] + R[u(t)] + N[u(t)] = g(t)\,,$$

$$\frac{d^{k-1}}{dt^{k-1}}u(0) = c_{k-1}, \qquad k = 1\,,2\,,3\,.\,.\,. \tag{1}$$

where, $L = \frac{d^k}{dt^k}$ is the highest derivative appear in the equation, R, N symbolize to a linear and nonlinear terms respectively , $g(t)$ is represents the no homogeneous term.

To solve Eq.(1) by VIM the below variational iteration formula can be obtained :

$$\Big\{ u_{n+1}(t) = u_n(t) + \int_0^t \lambda \{L[u_n(s)] + \overbrace{R[\tilde{u}_n(s)]} + \overbrace{N[\tilde{u}_n(s)]}\}ds \tag{2}$$

where λ is called a general Lagrange's multiplier [7] which one can be identified optimally via vartiational theory. $\overbrace{R[\tilde{u}_n(s)]}$ and $\overbrace{N[\tilde{u}_n(s)]}$ are considered as restricted variation [7] , i.e., $\delta\,\overbrace{R[\tilde{u}_n(s)]} = 0\,, \delta\,\overbrace{N[\tilde{u}_n(s)]} = 0.$

Calculating variation with respect to u_n,

$$\delta u_{n+1}(t) = \delta u_n(t) + \delta \int_0^t \lambda \left\{ L[u_n(s)] + \overbrace{R[\tilde{u}_n(s)]} + \overbrace{N[\tilde{u}_n(s)]} \right\} ds, \tag{3}$$

$$\delta u_{n+1}(t) = \delta u_n(t) + \delta \int_0^t \lambda \{L[u_n(s)]\}ds\,. \tag{4}$$

Therefore, the Lagrange multiplier, can be identified as [23].

$$\lambda = (-1)^k \frac{(s-t)^{(k-1)}}{(k-1)!} \tag{5}$$

Substituting Eq.(5) into Eq.(2) we get :

$$\left\{u_{n+1}(t) = u_n(t) + \int_0^t (-1)^k \frac{(s-t)^{(k-1)}}{(k-1)!} \{L[u_n(s)] + R[u_n(s)] + N[u_n(s)]\}ds \right. \tag{6}$$

Eq.(6) can be solved by using an initial approximation $u_0(x) = c_0 + c_1 t + c_2 \frac{t^2}{2!} + \cdots + c_{n-1} \frac{t^{k-1}}{(k-1)!}$,and then by iterating we get the appropriate approximate solution.

Now, to solve Eq.(1) by MVIM, Tamer A.Abassy [15] suggested effective modification by constructed an iteration formulation in the form

$$\left\{ \begin{array}{c} u_{n+1}(t) = u_n(t) + \int_0^t \lambda \{R[u_n(s) - u_{n-1}(s)] + \\ [G_n(s) - G_{n-1}(s)] - (g_{nk}s^{nk} + g_{nk+1}s^{nk+1} + \cdots + g_{nk+k-1}s^{nk+k-1})\}ds \end{array} \right. \tag{7}$$

where $G_n(t)$ is a polynomial function of degree $(kn + k - 1)$ and the relation $N[u_n(t)] = G_n(t) + O(t^{k(n+1)})$ is used to calculate its valued. g_n can be obtained by use Taylor's series expansion of $g(t)$ where $g(t) = \sum_{n=0}^{\infty} g_n t^n$.

The iteration formula defined in (3) can be iteratively solved using

$u_{-1}(t) = 0,$

$u_0(t) = \sum_{i=0}^{k-1} \frac{c_i}{i!} t^i,$

The approximate solution for Eq.(1). can be obtained approximately in the form

$$u(t) \cong u_n(t) \tag{8}$$

where the subscript n refers to the final iteration step.

MVIM can be easily used to solve k^{th} order fuzzy differential equation by using the following iteration formula

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$$
\begin{cases}
\underline{y}_{n+1}(t,r) = \underline{y}_n(t,r) + \int_0^t (-1)^k \frac{(s-t)^{(k-1)}}{(k-1)!} \{R[\underline{y}_n(s,r) - \underline{y}_{n-1}(s,r)] + \\
[\underline{G}_n(s,r) - \underline{G}_{n-1}(s,r)] - (g_{nk}s^{nk} + g_{nk+1}s^{nk+1} + \cdots + g_{nk+k-1}s^{nk+k-1})\}ds, \\
\\
\overline{y}_{n+1}(t,r) = \overline{y}_n(t,r) + \int_0^t (-1)^k \frac{(s-t)^{(k-1)}}{(k-1)!} \{R[\overline{y}_n(s,r) - \overline{y}_{n-1}(s,r)] + \\
[\overline{G}_n(s,r) - \overline{G}_{n-1}(s,r)] - (g_{nk}s^{nk} + g_{nk+1}s^{nk+1} + \cdots + g_{nk+k-1}s^{nk+k-1})\}ds,
\end{cases}
\quad (9)
$$

Where, $\underline{y}_0(t,r) = \sum_{i=0}^{k-1} \frac{\underline{c}_i}{i\,!} t^i$ and $\overline{y}_0(t,r) = \sum_{i=0}^{k-1} \frac{\overline{c}_i}{i\,!} t^i$ represent the lower and upper fuzzy initial value respectively. The notations $R[\underline{y}_n(s,r)]$ and $R[\overline{y}_n(s,r)]$ symbolize the linear terms. $\underline{G}_n(t,r)$ and $\overline{G}_n(t,r)$ are obtained from the relations $N[\underline{y}_n(t,r)] = \underline{G}_n(t,r) + O(t^{k(n+1)})$ and $N[\overline{y}_n(t,r)] = \overline{G}_n(t,r) + O(t^{k(n+1)})$, and g_n is obtained by Taylor's series expansion of $g(t)$,where $g(t) = \sum_{n=0}^{\infty} g_n t^n$

3. Numerical examples

In this section the MVIM is explained by solving two test examples of fuzzy differential equations . The required equations are formulated in accordance with what was done in Section 3 to get the approximate solution. By using the following formula the error is calculated:

$$\overline{E}(t,r) = \left|\overline{Y}(t,r) - \overline{y}(t,r)\right|,$$
$$\underline{E}(t,r) = \left|\underline{Y}(t,r) - \underline{y}(t,r)\right|.$$

Example (3.1): consider the following fuzzy initial value problem

$y'(t) - y(t) = \sin(t)\,, \qquad t \in [0,1]$

$\tilde{y}(0) = (0.96 + 0.04r, 1.01 - 0.01r)$ (10)

with the exact fuzzy solution:

$$\begin{cases} \underline{Y}(t,r) = (0.04r + 1.46)e^{t} - 0.5\,sin(t) - 0.5\cos(t) \\ \overline{Y}(t,r) = (1.51 - 0.01r)e^{t} - 0.5\,sin(t) - 0.5cos(t) \end{cases} \quad (11)$$

Now, to solving Eq.(10) by (MVIM) we have $R[\underline{y}(t)] = -\underline{y}(t)$, $R[\overline{y}(t)] = -\overline{y}(t)$ and $N[\underline{y}(t)] = 0$, $N[\overline{y}(t)] = 0$, $k = 1$ which leads to $\lambda = -1$, $g(t) = \sin(t)$.

Hence, through these facts and based on what was presented in section 3. MVIM approximate solution by use the following iteration formula can obtained:

$$\begin{cases} \underline{y}_{n+1}(t,r) = \underline{y}_n(t,r) - \int_0^t \{R\,[\,\underline{y}_n(s,r) - \underline{y}_{n-1}(s,r)] - (g_n s^n)\}ds, \\ \overline{y}_{n+1}(t,r) = \overline{y}_n(t,r) - \int_0^t \{R[\,\overline{y}_n(s,r) - \overline{y}_{n-1}(s,r)] - (g_n s^n)\}ds. \end{cases} \quad (12)$$

where $\underline{y}_{-1}(t,r) = \overline{y}_{-1}(t,r) = 0$ and g_n calculated by using Taylor's series expansion of $(\sin(t))$ at $t = 0$

$$\big(sin(t)\big) = \sum_{n=0}^{\infty} g_n t^n$$

If we begin with

$$\begin{cases} \underline{y}_0(t,r) = 0.96 + 0.04r \\ \overline{y}_0(t,r) = 1.01 - 0.01r \end{cases}$$

then the approximation solution $y(t,r) = (\underline{y}(t,r), \overline{y}(t,r))$, of Example (4.1) at $t = 0.01$ and $r \in [0,1]$ are shown in Table 1 and Figure 2.

Table 1.Upper and Lower MVIM Approximate Solution of Example (3.1). at $t = 0.01$

R	$\underline{Y}$(EXACT)	$\underline{Y}$(MVIM)	$\overline{Y}$(EXACT)	$\overline{Y}$(MVIM)	$\underline{Y}$(MVIM ERRO	$\overline{Y}$(MVIM ERRO
0	0.969698327	0.969698327	1.020200835	1.020200835	0.000000000	0.000000000
0.2	0.977778728	0.977778728	1.018180735	1.018180735	0.000000000	0.000000000
0.4	0.985859129	0.985859129	1.016160634	1.016160634	0.000000000	0.000000000
0.6	0.993939531	0.993939531	1.014140534	1.014140534	0.000000000	0.000000000
0.8	1.002019932	1.002019932	1.012120434	1.012120434	0.000000000	0.000000000
1	1.010100333	1.010100333	1.010100333	1.010100333	0.000000000	0.000000000

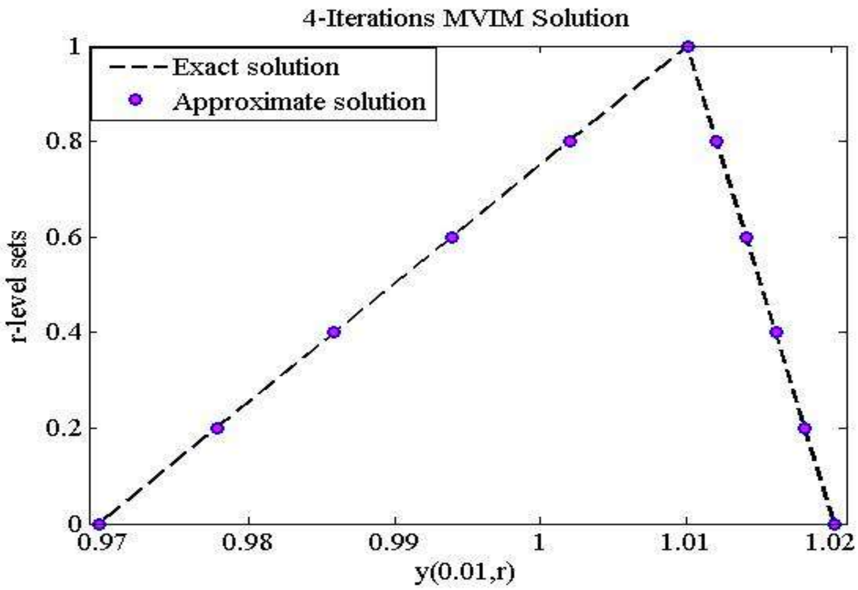

Figure 2. MVIM approximate solution of (10) at $t = 0.01$ and $r \in [0, 1]$

According to Tables 1 and Figure 2, when using MVIM we omit the highest derivative of the equation (10) and the function sin(x) is not used for each step but parts of its Taylor series are used based on what is described in section 3. By taking 4-iterations MVIM the approximate solutions of (10) for all $t \in [0,1]$ and $t \in [0,1]$ are obtained, where $\underline{y}(t,r)$, $\overline{y}(t,r)$ is the lower and the upper MVIM approximate solutions respectively and $\underline{Y}(t,r), \overline{Y}(t,r)$ is the lower and the upper exact solution respectively.

Example (3.2): let us have the following FDEs

$$y'(t) = y^2(t), \qquad t \in [0,1]$$

$$\tilde{y}(0) = (0.4 + 0.2r, 0.9 - 0.3r) \tag{13}$$

with the exact fuzzy solution [24]:

$$\begin{cases} \underline{Y}(t,r) = \frac{0.4+0.2r}{1-(0.4+0.2r)t} \\ \overline{Y}(t,r) = \frac{0.9-0.3r}{1-(0.9-0.3r)t} \end{cases} \tag{14}$$

To solving Eq.(13) by (MVIM) we have $R[\underline{y}(t)] = 0$, $R[\overline{y}(t)] = 0$ and $N[\underline{y}(t)] = \underline{y}^2(t)$, $N[\overline{y}(t)] = \overline{y}^2(t)$, $k = 1$ which leads to $\lambda = -1, g(t) = 0$.

Therefore, the next iteration formula is used to compute approximate MVIM solution :

$$\begin{cases} \underline{y}_{n+1}(t,r) = \underline{y}_n(t,r) - \int_0^t \{[\underline{G}_n(s,r) - \underline{G}_{n-1}(s,r)]\}ds, \\ \overline{y}_{n+1}(t,r) = \overline{y}_n(t,r) - \int_0^t \{[\overline{G}_n(s,r) - \overline{G}_{n-1}(s,r)]\}ds. \end{cases} \tag{15}$$

where $\underline{G}_{-1}(t,r) = \overline{G}_{-1}(t,r) = 0$,and $\underline{G}_n(t,r), \overline{G}_n(t,r)$ are obtained from the formula

$$(\underline{y}_n(t,r))^2 = \underline{G}_n(t,r) + O(t^{n+1}), (\overline{y}_n(t,r))^2 = \overline{G}_n(t,r) + O(t^{n+1}),$$

If we start with

$$\begin{cases} \underline{y}_0(t,r) = 0.4 + 0.2r \\ \overline{y}_0(t,r) = 0.9 - 0.3r, \end{cases}$$

then the approximation solution $y(t,r) = (\underline{y}(t,r), \overline{y}(t,r))$,, of Example (4.2) at $t = 0.01$ and $r \in [0,1]$ are shown in Table 2-3 and Figure 3.

Table 2. Lower MVIM approximate solution $\underline{y}(t,r)$ of example 4.2 at $t = 0.01$ and $r \in [0,1]$

R	$\underline{Y}$(EXACT)	$\underline{Y}$(MVIM)	$\underline{Y}$(VIM)	$\underline{Y}$(MVIM)ERROR	$\underline{Y}$(VIM)ERROR
0	0.401606425	0.401606425	0.401606425	0.000000000	0.000000000
0.2	0.441944556	0.441944556	0.441944556	0.000000000	0.000000000
0.4	0.482315112	0.482315112	0.482315112	0.000000000	0.000000000
0.6	0.522718134	0.522718134	0.522718134	0.000000000	0.000000000
0.8	0.563153660	0.563153660	0.563153660	0.000000000	0.000000000
1	0.603621730	0.603621730	0.603621730	0.000000000	0.000000000

Table 3. Upper approximate solution $\overline{y}(t,r)$ of example 4.2 at $t = 0.01$ and $r \in [0,1]$

R	$\overline{Y}$(EXACT)	$\overline{Y}$(MVIM)	$\overline{Y}$(VIM)	$\overline{Y}$(MVIM)ERROR	$\overline{Y}$(VIM)ERROR
0	0.908173562	0.908173562	0.908173562	0.000000000	0.000000000
0.2	0.847115772	0.847115772	0.847115772	0.000000000	0.000000000
0.4	0.786131828	0.786131828	0.786131828	0.000000000	0.000000000
0.6	0.725221595	0.725221595	0.725221595	0.000000000	0.000000000
0.8	0.664384940	0.664384940	0.664384940	0.000000000	0.000000000
1	0.603621730	0.603621730	0.603621730	0.000000000	0.000000000

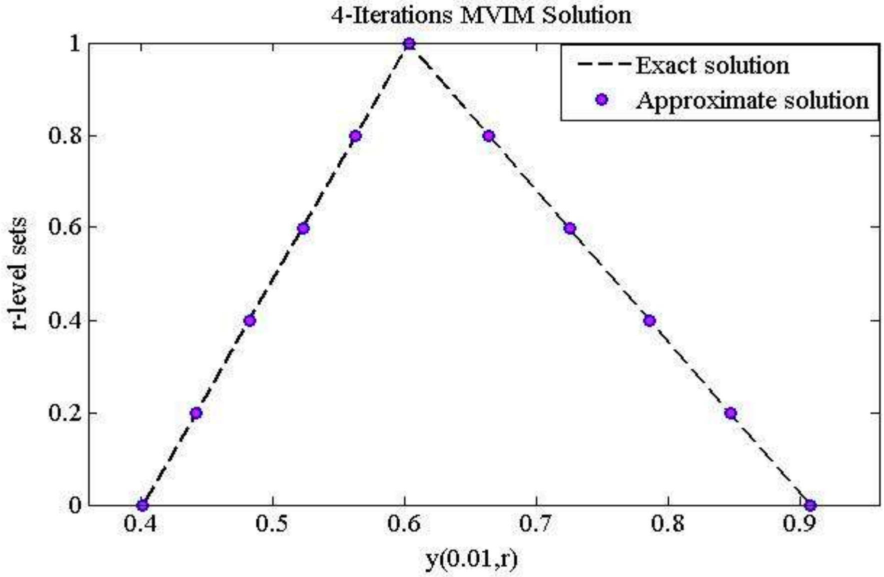

Figure 3. MVIM approximate solution of (13) at $t = 0.01$ and $r \in [0,1]$

From the above results of Example (4.2) are shown in Table 2-3, which obtained by 4-iterations MVIM. The MVIM and VIM has the same approximate solutions, but when using MVIM we can take a large number of iterations with complete comfort without complications, and thus we can reach a very accurate and perfect approximation to the exact solution, while when using VIM there is great difficulty if the number of iterations increases.

Conclusion

In this research study, modified variational iteration method (MVIM) has been applied to compute the approximate solution of fuzzy differential equations (FDEs) and how MVIM works is discussed. The MVIM gives out a corrections functional that act as a series of approximate solutions that gradually get closer to the exact solution. The results are obtained by MVIM using Mathcad 15 and Matlap20a programs and after analyzing them led us to the following important conclusion that (MVIM) is documented, effective and convergent. Also MVIM is faster than VIM and save time. As a future work we suggest fuzzy delay differential equations.

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