Coalition Graphs of Complete Bipartite Graphs

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ABSTRACT

A coalition in a graph G= (V, E) consists of two disjoint, non-dominating vertex subsets whose union forms a dominating set, and a coalition graph is constructed from a coalition partition by representing each part as a vertex and connecting two parts if they form a coalition. This paper provides a comprehensive structural analysis of coalition graphs derived from complete bipartite graphs $K_{r,s}$, a fundamental graph class with broad applications in resource allocation, scheduling, and two-layered network systems. We completely characterize all possible coalition graphs for star graphs $K_{1,s}$, proving that they are strictly limited to three isomorphism types: K_1, K_2, or the disjoint union $K_1 \cup K_2$, thereby revealing the restrictive coalition behavior imposed by the hub-and-spoke topology. For general complete bipartite graphs $K_{r,s}$, we establish several key structural results: the coalition number $c(K_{r,s})$ is exactly $r+s$, attained by the singleton partition; the independence number of any coalition graph derived from $K_{r,s}$ is at least $\max\{r,s\}$; and the maximum degree satisfies $\Delta(CG(K_{r,s};\pi))\leq\max\{r,s\}+1$ for any coalition partition π. These findings provide a complete theoretical description of coalition structures within bipartite frameworks, extending the known families of coalition graphs beyond paths, cycles, and trees. By bridging structural properties with the membership requirements of complete bipartite graphs, this work establishes a robust mathematical foundation for addressing the inverse problem of coalition graphs and opens new directions for exploring higher-order multipartite structures and their computational complexities.

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1. INTRODUCTION

Graphs, being mathematical structures, model relationships and interactions in several disciplines such as computer science, biology, social networks, communication systems, etc. In graph theory, thus, the study of dominating sets and partitions has been essentially targeted at realizing the ability of a subset of vertices to control and cover. One interesting extension to this concept is when two or more subsets of vertices join their powers and eventually form what is known as a coalition.

The notion of a coalition in a graph was first introduced to capture cooperative interactions between vertex subsets that alone are not sufficient to dominate the graph but become effective when combined [1]. This conceptual framework contains applications in network security, group1 decision-making, and distributed systems. For instance, it is conceivable that two subgroups inside a social network might be incapable of influencing the entire network, but joining together, they achieve full influence. This can be modeled quite naturally using coalition structures in graphs. A coalition partition (or c-partition) expresses formally how the vertices in a graph can be arranged so that any group is either a dominating single vertex (usually representing an entity with full access or influence) or a non-dominating set that joins with another to perform domination. The relations between these sets are represented in a coalitional graph structure, where the vertices stand for subsets in the c-partition and the edges for successful cooperation, that is, and the formation of a coalition. Coalition graphs were initially studied in the context of simple graph families such as paths, cycles, and trees [2]. These foundational works revealed interesting structural properties For example, it was shown that the families of paths and cycles yield only finitely many distinct coalition graphs. In contrast, the broader class of trees can give rise to an infinite variety of coalition graphs due to their diverse structural configurations. These results provide a fundamental basis for understanding how graph structure impacts coalition behavior.

Recent extensions of coalition concepts have emerged, broadening the classical framework to include variations such as strong, perfect, edge-based, and total coalition structures. Mojdeh and Samadzadeh [3] introduced the notion of *perfect coalitions*, where the union of two non-dominating sets yield a perfect dominating set. Gol Mohammadi et al. [4] defined *strong coalitions*, adding degree-based conditions to domination. Edge-based coalition structures were studied by Mojdeh and Masoumi [5] that were focusing on edge domination and partitioning. In 2024, Henning and Jogan [6] characterized graphs based on *total coalition numbers*, where the union of disjoint sets achieves total domination. Finally, bipartite graphs were analyzed in the context of star discoloring and coloring bounds by Gaur et al. [7], providing algorithmic insights for related structural properties.

However, many practical networks do not fall into the categories of paths, cycles, or trees. In many applications, especially bipartite systems (e.g., resource-assignment problems, team-task scheduling, or peer-to-peer collaboration networks), it is more natural to model the system as a star or a complete bipartite graph. In a star graph, a central node interacts with all others, which is a common topology in network hubs and central server architectures. In complete bipartite graphs, two disjoint sets of vertices have full interconnection across the partition, modeling scenarios such as job assignment from workers to tasks or two-layered influence networks.

Despite their relevance, coalition graphs for stars and complete bipartite graphs have not yet been fully explored in the literature. The goal of this paper is to fill this gap. Specifically, we aim to:

- Characterize all possible coalition graphs that can be generated from c-partitions of star graphs $K1,n$.
- Investigate structural properties and possible bounds on coalition graphs derived by complete bipartite graphs Kr,s.
- Provide concrete examples and diagrams to visualize the coalition behavior in these graphs.

Understanding these coalition graphs can have significant theoretical and practical implications.

From a theoretical perspective, this extends the known families of coalition graphs and opens the door to further generalizations. From a practical standpoint, analyzing coalition behaviors in star and bipartite graphs can guide the design of more robust and cooperative network systems.

The remainder of this paper is organized as follows: Section 2 presents the formal definitions and background needed for coalition graphs. Section 3 explores the coalition graphs of star graphs and provides a complete classification. Section 4 examines complete bipartite graphs, including examples, structural bounds, and open questions. Section 5 offers concluding remarks and directions for future research.

2. PRELIMINARIES

We now introduce the notions central to our study.

Definition 2.1. [open and closed neighborhood][8] Let G = (V, E) be a finite, simple and undirected graph. The open neighborhood of v ∈V , denoted by N(v), is defined as

$N(v) = \{u \in V \mid uv \in E\}$,

that is, the set of all vertices adjacent to v. The closed neighborhood of v, denoted by N[v], is defined as $N[v] = N(v) \cup \{v\}$.

Intuitively, the open neighborhood N(v) represents the vertices that are directly influenced or reached by v through adjacency, while the closed neighborhood N[v] includes v itself in addition to all its neighbors. The distinction between open and closed neighborhoods plays a fundamental role in domination theory.

A vertex v dominates all vertices in its closed neighborhood N[v], and a set S ⊆V is a dominating set of G if and only if

$$\bigcup_{v \in S} N[v] = V.$$

Hence, domination is inherently defined in terms of closed neighborhoods.

In contrast, open neighborhoods are useful for describing adjacency structure and degree-related properties. In particular, the degree of a vertex v satisfies deg(v) = |N(v)|, and the size of N[v] is

deg(v) + 1.
Let $G = (V, E)$ be a finite, simple, undirected graph. For a vertex $v \in V$, let $N(v)$ denote theopen neighborhood of v, and $N[v] = N(v) \cup v$ the closed neighborhood. Moreover, the maximum degree of a graph G, denoted $\Delta(G)$, is defined as the largest vertex degree found in G.
$\Delta(G) = \max\{\deg(v) : v \in V\}$

Recall that the degree of a vertex v, deg(v), is the number of edges connected to v, or the number of neighbors v has.
A set $S \subseteq V$ is a dominating set if$\bigcup_{v \in S} N[v] = V$. Graph domination is a fundamental concept in graph theory and has been widely studied for its applications in network coverage, influence propagation, facility location problems, and many other areas. A dominating set provides a measure of how a subset of vertices can effectively reach or influence the entire graph through direct or indirect adjacency.
Throughout this paper, neighborhoods are used to determine whether a vertex set (or the union of two vertex sets) dominates the graph, which is essential in defining coalitions and constructing coalition graphs. In particular, two disjoint sets V1 and V2 form a coalition precisely when the union of their closed neighborhoods covers the entire vertex set of the graph.
Definition 2.2. [independent set][9] An independent set (also called a stable set) in a graph $G = (V, E)$ is a set of vertices $S \subseteq V$ such that there are no edges between any two vertices in S.
In other terms, for every pair of distinct vertices $u, v \in S$, the edge $(u, v) \notin E$.
Given a graph G, the vertex independence number $\alpha(G)$ is the size of the largest independent set in the graph. Formally,
$\alpha(G) = \max\{|S| : S \subseteq V, S \text{ is an independent set in } G\}$.
Definition 2.3. [bipartite graph][1, 9] A bipartite graph is a graph $G = (V, E)$ whose vertex set V can be partitioned into two disjoint subsets A and B such that every edge in E has one endpoint in A and the other in B. That is, no edge connects two vertices in the same subset.

A complete bipartite graph, denoted by $K_{r,s}$, is a bipartite graph in which every vertex in A is connected to every vertex in B, where $|A| = r$ and $|B| = s$.
Definition 2.4. [coalition][9] Let $G = (V, E)$ be a graph. Two nonempty, disjoint vertex sets $V_1, V_2 \subseteq V$ are said to form a coalition in G if the following conditions hold:
1. V1 is not a dominating set of G;
2. V2 is not a dominating set of G;
3. the union $V_1 \cup V_2$ is a dominating set of G, that is,
$$\bigcup_{v \in V_1 \cup V_2} N[v] = V.$$
In this case, the ordered pair (V_1, V_2) (or equivalently, the unordered pair $\{V_1, V_2\}$) is called a coalition in G.
This definition captures the essence of cooperative interaction. The union of two non-dominating sets can yield a dominating set. It mimics real-world behavior where individual agents (nodes) may not achieve coverage or influence alone but can do so through strategic partnerships.
Definition 2.5. [coalition partition][1] A coalition partition (c-partition) of G is a partition $\pi = V_1, \ldots, V_k$ of the vertex set such that each V_i is either:
• a singleton set v where v is a universal vertex (i.e., a vertex of degree $n-1$ and thus dominating on its own), or
• a set which is not a dominating set but forms a coalition with some other V_j in the partition.
The definition of c-partition ensures that all parts of the partition play a role in covering the graph either individually or through partnerships. The requirement that singleton dominating sets be universal is essential, as otherwise multiple singleton dominating sets could trivialize the structure of the coalition graph.

Given a c-partition π, the coalition graph $CG(G, \pi)$ has vertex set $\{V_1, \ldots, V_k\}$, with an edge V_iV_j if and only if V_i and V_j form a coalition in G.
The coalition graph abstracts the partnership relationships within the partition. Vertices of the coalition graph represent parts of the partition of G, and edges signify successful cooperation.
Notably, singleton dominating sets become isolated vertices in the coalition graph, since they require no coalition to dominate the graph. The structure of the coalition graph depends heavily on the topology of G and the way the c-partition is constructed. For instance, if G has a universal vertex, then the coalition graph will necessarily have an isolated vertex. If G has many vertices of low degree, coalition formation becomes a critical mechanism for achieving domination.
Definition 2.6. [complete graph] The complete graph K_n is a graph with n vertices in which every pair of distinct vertices is connected by an edge. In particular:
• K_1 is the graph with a single vertex and no edges.
• K_2 is the graph with two vertices and one edge between them.

Definition 2.7. [star graph] A star graph is a tree with one central vertex connected to all other vertices, which each have a degree of 1. It's a specific type of graph where one node (the center) is connected to all other nodes, which are not connected to each other.

These graphs frequently appear as building blocks or subgraphs in coalition graph structures.

Let us consider a few more observations that help in analyzing coalition graphs:

- If a vertex in G is isolated, it cannot be part of any dominating set. Thus, a graph containing isolated vertices cannot admit a valid c-partition.
- The coalition graph CG(G, π) reflects domination properties indirectly; in fact, the presence or absence of an edge between Vi and Vj encodes whether the union of these sets dominates G.
- The maximum degree of a vertex in a coalition graph can be bounded using the maximum degree $\Delta(G)$ of the original graph, as coalition possibilities depend on the intersection of neighborhood.

Definition 2.8. [*coalition number*][1] The coalition number of a graph G, denoted by $c(G)$, is the maximum possible cardinality of a c-partition of G. Equivalently, it is the maximum number of parts in any c-partition of G.

The coalition number measures how finely the vertex set of a graph G can be partitioned while preserving the coalition property. In the context of coalition graphs, an independent set corresponds to a collection of parts in the c-partition that do not form coalitions with each other. The *greatest independent set* refers to an independent set of maximum size; thus, the terms *greatest independent set* and *maximum independent set* are equivalent in meaning, although we use the latter for consistency with standard graph-theoretic terminology.

A greatest c-partition determines the vertex set of a coalition graph with the largest possible number of vertices. Properties such as the independence number of the coalition graph describe how many parts of the partition can remain non-cooperative. Hence, the coalition number concerns the size of the partition itself, while the independence number of the coalition graph concerns the interaction structure among the parts of that partition.

In later sections, we will use these foundational ideas to develop and analyze coalition graphs derived from specific classes of graphs, namely star graphs and complete bipartite graphs. But, the simplicity of their structure offers opportunities for complete classification.

3. Coalition Graphs of Complete Bipartite Graphs

We investigate the coalition structure of complete bipartite graphs $K_{r,s}$. Without loss of generality, we assume the vertex set is partitioned into two independent sets A and B with $|A| = r$ and $|B| = s$, where $r \leq s$ unless stated otherwise. This ordering is convenient for stating bounds, but we will explicitly note when a result depends on it.

Maximum degree of coalition graphs

We begin by bounding the maximum degree of the coalition graph of a complete bipartite graph.

Theorem 3.1. Let $K_{r,s}$ be a complete bipartite graph with partite sets A and B of sizes r and s, respectively, and let π be a c-partition of $K_{r,s}$. Then

$\Delta(CG(K_{r,s}; \pi)) \leq \max\{r, s\} + 1.$

Proof. Let V_i be a part of the c-partition π. If V_i is a dominating set of $K_{r,s}$, then V_i must contain vertices from both A and B, and hence V_i corresponds to an isolated vertex in $CG(K_{r,s}; \pi)$.Suppose now that V_i is not a dominating set. Then V_i is contained entirely in one of the partite sets, say $V_i \subseteq A$ or $V_i \subseteq B$. Without loss of generality, assume $V_i \subseteq A$. In order to form a coalition with V_i, any other part V_j must contain at least one vertex of B. Since $|B| = s$, there are at most s such parts, together with at most one additional part contained in A. Therefore,

$\deg_{CG}(V_i) \leq s + 1 \leq \max\{r, s\} + 1.$

Hence, as claimed. $\Delta(CG(K_{r,s}; \pi)) \leq \max\{r, s\} + 1,$

Corollary 3.1. Let $K_{1,s}$ be a star graph. Then any coalition graph CG($K_{1,s}$; π) is isomorphic to K_1, K_2, or $K_1 \cup K_2$.Proof. Let V ($K_{1,s}$) = {v_0, v_1, v_2, . . . , v_s} where v_0 is the center and v_i for i = 1, . . . , s are leaves of $K_{1,s}$. The set {v_0} dominate $K_{1,s}$, but the sets {v_j} for j = 1, . . . , s are not dominating set, individually. Moreover, the union of V_i and V_j is a dominating set, if $|V_i \cup V_j| = s$. Hence, $CG(K_{1,s}; \pi)$ is isomorphic to K_1, K_2, or $K_1 \cup K_2$.

Theorem 3.2. Let $K_{r,s}$ be a complete bipartite graph with partite sets A and B of sizes r and s, respectively. For any c-partition π of $K_{r,s}$, $\alpha(CG(K_{r,s}; \pi)) = s.$

Proof. Let $\pi = \{V1, V2, \ldots, Vk\}$ be a c-partition of Kr,s. For any part $Vi \in \pi$, the following hold:
Case 1. If $Vi \subseteq A$ and $|Vi| < r$, then Vi is not a dominating set.
Case 2. If $Vi \subseteq B$ and $|Vi| < s$, then Vi is not a dominating set.
Case 3. If Vi contains vertices from both A and B, then Vi is a dominating set.
Since Kr,s has no universal vertex for $r, s \geq 2$, every part of a c-partition must be a non-dominating set and therefore must be a proper subset of either A or B.Assume without loss of generality that $r \leq s$. Let
$\pi = \{\{b1\}, \{b2\}, \ldots, \{bs\}, \{a\}\}$,
where $b1, \ldots, bs \in B$ and $a \in A$. Then $CG(Kr,s; \pi)$ is a star with center $\{a\}$, and
$S = \{\{b1\}, \{b2\}, \ldots, \{bs\}\}$
is an independent set of maximum size. Hence $\alpha(CG(Kr,s; \pi)) = s$.

Remark 3.1. If we drop the assumption $r \leq s$, then the lower bound becomes $\alpha(\mathrm{CG}(Kr,s, \pi)) \geq \max\{r, s\}$. The exact value depends on the chosen partition π; for the singleton partition it equals $\max\{r, s\}$, but for other partitions it may be larger.

Theorem 3.3. Let Kr,s be a complete bipartite graph with partite sets A and B of sizes r and s, respectively. The coalition number of Kr,s is $r + s$.

Proof. The greatest c-partition π of a complete bipartite graph Kr,s is given by
$\pi = \{V1, \ldots, Vr, W1, \ldots, Ws\}$,
where $Vi = \{vi\}$ for $i = 1, \ldots, r$ are the vertices of A and $Wi = \{wi\}$ for $i = 1, \ldots, s$ are the vertices of B. No set Vi is a dominating set, nor is any set Wj. However, $Vi \cup Wj$ for any i and j is a dominating set. Hence, the coalition number of Kr,s is $r + s$.

4. Examples and Illustrations

In this section, we present illustrative examples of coalition graphs formed from specific c-partitions of well-known graphs. These examples are supported by diagrams to provide intuitive insight into the structure and interaction of coalition sets. The coalition graph associated with each example helps visualize how subsets collaborate to dominate the original graph.

Example 4.1. Let $G = K1,3$ be a star graph with center $v0$ and leaves $v1$, $v2$, and $v3$ (see Figure1). Consider the c-partition $\pi = \{V0 = \{v0\}, V1 = \{v1, v2\}, V2 = \{v3\}\}$. Here, $V0$ is a singleton dominating set and thus becomes an isolated vertex in the coalition graph. The sets $V1$ and $V2$ are not dominating individually, but together their union dominates the entire graph: $N[v1] \cup N[v2] \cup N[v3]$ includes the center $v0$. Hence, they form a coalition. The resulting coalition graph is $K1 \cup K2$
(see Figure 2).

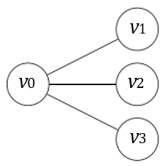

Figure 1. Star graph $K1$, 3 with center $V0$ and leaves $v1$, $v2$ and $v3$

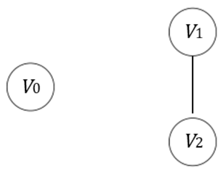

Figure 2. Coalition graph $CG(G,\pi)$: K_1:K_2

Example 4.2. Let $G = K2,2$ with partite sets $A = \{a1, a2\}$ and $B = \{b1, b2\}$ (see Figure 3). Now, consider the partition $\pi = \{\{a1\}, \{a2\}, \{b1, b2\}\}$. In this partition, $\{a1\}$ and $\{a2\}$ are not dominating sets, but $\{b1, b2\}$ is a dominating set. Hence, $\{b1, b2\}$ appears as an isolated vertex in the coalition graph, while $\{a1\}$ and $\{a2\}$ do not form a coalition with each other. Therefore, the coalition graph is $K2 \cup K1$ (see Figure 4). Also, $\Delta(CG(K2,2; \pi)) = 1$.

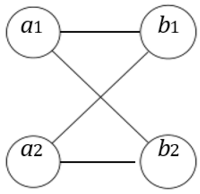

Figure 3. Complete bipartite graph $G=K_{2,2}$

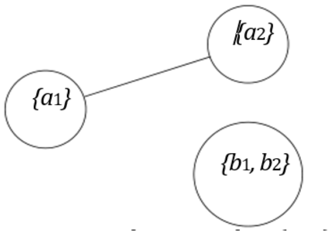

Figure 4. Coalition graph $CG(G,\pi)$: K_1:K_2

Example 4.3. Let $G = K2,3$ with partite sets $A = \{a1, a2\}$ and $B = \{b1, b2, b3\}$ (see Figure 5). Let us consider the partition $\pi = \{\{a1\}, \{a2\}, \{b1, b2\}, \{b3\}\}$. The sets $\{a1\}$ and $\{a2\}$ are not dominating individually, nor are any of the B sets. However, $\{a1\} \cup \{b1, b2\}$ and $\{a2\} \cup \{b1, b2\}$ both dominate G. The set $b3$ only connects to $a1$ and $a2$, and must participate in further coalitions (see Figure 6).

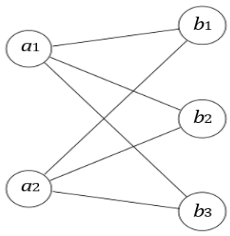

Figure 5. Complete bipartite graph $G=K_{2,3}$

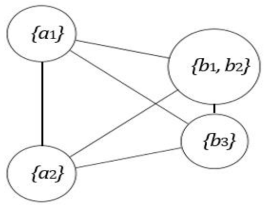

Figure 6. Coalition graph $CG(G,\pi)$ includes edges from $\{a_1\}$ and $\{a_2\}$ to both $\{b_1, b_2\}$ and $\{b_3\}$

Example 4.4. Let $G = K3,4$ be a complete bipartite graph with partite sets $A = \{a1, a2, a3\}$ and $B = \{b1, b2, b3, b4\}$. Consider the c-partition

$\pi = \{\{a1\}, \{a2\}, \{a3\}, \{b1, b2\}, \{b3\}, \{b4\}\}$.

Each part of π is a non-dominating set, since every part is a proper subset of either A or B. Moreover, the union of any singleton set $\{ai\}$ with any part containing vertices from B forms a dominating set of G. Hence, each $\{ai\}$ forms a coalition with each of $\{b1, b2\}$, $\{b3\}$, and $\{b4\}$.

However, no two parts contained entirely in A form a coalition, and no two parts contained entirely in B form a coalition. Therefore, the coalition graph $CG(G, \pi)$ is isomorphic to the complete bipartite graph $K3,3$.

This example demonstrates that coalition graphs of complete bipartite graphs can exhibit rich structures beyond stars and disjoint unions, and in particular may themselves be complete bipartite graphs, (see Figure 7)

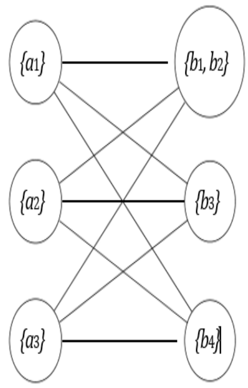

Figure 7. Coalition graph $CG(G,\pi)$ for the graph $G=K_{3,4}$

These examples demonstrate how different choices in partitioning the vertices of a graph can lead to different coalition structures. In particular, the role of domination and the graph topology play a key role in determining the nature of coalitions. In the next section, we formally characterize the structure of coalition graphs for star and complete bipartite graphs and provide supporting theorems with proofs.

5. CONCLUSION

This paper provides a rigorous structural analysis of coalition graphs G_c associated with complete bipartite graphs K_{r,s}, a class of graphs fundamental to modeling complex interaction networks. Our primary contribution is the complete characterization of coalition graphs for star graphs K_{1,s}, where we proved that such structures are strictly limited to K_1, K_2, or the disjoint union K_1 \cup K_2.
Furthermore, we established a definitive result for the general bipartite case, proving that the coalition number C(K_{r,s}) is precisely r + s. This finding, coupled with the established tight bounds on the degree and independence number \alpha(G_c), offers a deep insight into the internal symmetry and membership dynamics of coalitions within bipartite frameworks. By bridging the gap between theoretical properties and the membership requirements of K_{r,s}, this work provides a robust mathematical foundation that paves the way for solving the inverse problem of coalition graphs. These results not only extend the existing literature on coalition theory but also set a new benchmark for exploring higher-order multipartite structures and their computational complexities.

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