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Homotopy Theory of Micro Topological Spaces

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ABSTRACT

This article looks at the link between homotopy theory and micro topological spaces, focusing on the new ideas that have come up: micro homotopy and micro homotopy equivalence. By extending classical homotopy theory to micro topological spaces, we capture more complex structural features at an infinitesimal scale. Micro homotopy is a continuous change deformation between maps that keeps the microstructure of the spaces that are involved. Two micro topological spaces are micro homotopy equivalent if there exist micro continuous maps between them whose compositions are micro homotopic to the respective identity maps. These ideas give us a better way to think about equivalence in topological analysis.

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1. INTRODUCTION

Homotopy theory is an essential aspect of algebraic topology that studies how functions, spaces, or structures can be continuously deformed. This is an excellent manner to group topological spaces based on their “shape” rather than their exact structure. This idea is very important for studying the fundamental attributes of spaces that don't change when they are transformed continuously; for additional details, see [1, 2, 3, and 4].

On the other hand, diverse topological spaces are continuously being developed by numerous topologists. In 1968, Chang [5] first addressed the concept of fuzzy topological spaces. Further, in 2011 Shabir and Naz [6] presented soft topological spaces that have been attracted by the group of investigators, for example, see [7],[8]. In 2013, Thivagar and Richard [9] first proposed the notion of nano topology. Through the year in 2019, Chandrasekar [10] introduced micro topological spaces, which are considered in this work. It generalises ordinary topological principles by enhancing the concept of open sets through frameworks like micro-topologies or nano-topologies. These spaces often arise in the study of expanded rough sets, nano structures, and their applications in logic and information systems and are extensively utilised by the group of specialists; see [11, 12, 13, 14, and 15]. In micro topological spaces, open sets can be created through more accurate approximations, like those made by equivalence relations. This makes it easier to study continuity, convergence, and homotopy in a more advanced way.

More importantly, in this work, the relationship between homotopy and micro topological spaces opens new research directions, allowing us better invariants and more techniques to use them, especially when classical topological tools don't work. The study of micro-continuous functions and their homotopies enhances classical theory, enabling the investigation of new forms of connectivity and deformation within generalised topological frameworks.

2. PRELIMINARIES

Definition (2.1): Presuming that $(\mathfrak{U},\mathfrak{R})$ is a space of approximation, whenever $\mathfrak{U}$ is the universe along with $\mathfrak{R}$ is a relation an of indiscernibility upon $\mathfrak{U}$, then we can define the approximation of $\mathfrak{L}_{\mathfrak{R}}$ - lower as well $\mathfrak{T}_{\mathfrak{R}}$ upper of a $\mathfrak{S} \subseteq \mathfrak{U}$ as follows.

$$\mathfrak{L}_{\mathfrak{R}}(\mathfrak{S}) = \bigcup_{q\in\mathfrak{S}} \{\mathfrak{R}(q) : \mathfrak{R}(q) \subseteq \mathfrak{S}\}$$

$$\mathfrak{T}_{\mathfrak{R}}(\mathfrak{S}) = \cup_{q\in\mathfrak{S}} \{\mathfrak{R}(q) : \mathfrak{R}(q) \cap \mathfrak{S} \neq \emptyset\}.$$

Here, $\mathfrak{R}(q)$ is the class of equivalence containing an element $q \in \mathfrak{S}$. The boundary area of $\mathfrak{S}$ is indicated as $\mathfrak{B}_{\mathfrak{R}}(\mathfrak{S})$ and defines as

$$\mathfrak{B}_{\mathfrak{R}}(\mathfrak{R}) = \mathfrak{T}_{\mathfrak{R}}(\mathfrak{S}) - \mathfrak{L}_{\mathfrak{R}}(\mathfrak{S}).$$

Definition (2.2): Let $(\mathfrak{U},\mathfrak{R})$ be a space of approximation while $\mathfrak{S} \subseteq \mathfrak{U}$. Then, the topology $\tau_{\mathfrak{R}}(\mathfrak{S}) = \{\mathfrak{U}, \emptyset, \mathfrak{L}_{\mathfrak{R}}(\mathfrak{S}), \mathfrak{T}_{\mathfrak{R}}(\mathfrak{S}), \mathfrak{B}_{\mathfrak{R}}(\mathfrak{S})\}$ over $\mathfrak{U}$ is identified as nano regarding $\mathfrak{S}$.

The space$(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{S}))$ is the nano topology and the components from $\tau_{\mathfrak{R}}(\mathfrak{S})$ are known nano open.

Definition (2.3): Given a nano topological space $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{S}))$, the micro topology $\mathcal{M}_{\mathfrak{R}}(\mathfrak{S})$ on $\mathfrak{U}$ by $\mathcal{M}$ where $\mu \notin \mathcal{M}_{\mathfrak{R}}(\mathfrak{S})$ is defined as

$$\mathcal{M}_{\mathfrak{R}}(\mathfrak{S}) = \{\mathcal{N} \cup (\grave{\mathcal{N}} \cap \mathcal{M}) : \mathcal{N}, \grave{N} \in \tau_{\mathfrak{R}}(\mathfrak{S})\}.$$

The triplet $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{S}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{S}))$ is referred to as spaces of micro topology. The components from $\mathcal{M}_{\mathfrak{R}}(\mathfrak{S})$ are known as micro open. It is clear that micro topology is a generalisation of nano topology, meaning it includes nano topology. So, every nano topology is micro topology.

Example (2.4): Assume that $\mathfrak{U} = \{i, j, k, \ell\}$ in addition to $\mathfrak{U}/_{\mathfrak{R}} = \{\{j\}, \{\ell\}, \{i, k\}\}$ and the subset $\mathfrak{S} = \{k, \ell\}$. So,

$$\tau_{\mathfrak{R}}(\mathfrak{S}) = \{\mathfrak{U}, \emptyset, \{\ell\}, \{i, k\}, \{i, k, \ell\}\}.$$

Suppose $\mathcal{M} = \{j\}$, then

$$\mathcal{M}_{\mathfrak{R}}(\mathfrak{S}) = \{\mathfrak{U}, \emptyset, \{j\}, \{\ell\}, \{i, k\}, \{j, \ell\}, \{i, j, k\}, \{i, k, \ell\}\}.$$

Definition (2.5): Let the spaces $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{S}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{S}))$ and $(\mathfrak{V}, \tau_{\mathfrak{R}}(\mathfrak{Y}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{Y}))$ be micro topology. The function

$$\mathcal{F} : \mathfrak{U} \to \mathfrak{V}$$

Is claimed micro-continuous whether the image of inverse for all micro open of $\mathfrak{V}$ is a micro open of $\mathfrak{U}$.

3. MAIN RESULTS

This section explains the micro homotopy and the several topological aspects that can be inferred from it.

Definition (3.1): Suppose $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{S}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{S}))$ and $(\mathfrak{V}, \tau_{\mathfrak{R}}(\mathfrak{Y}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{Y}))$ are micro topological spaces. Let

$$\mathcal{H} : \mathfrak{U} \times \mathrm{I} \to \mathfrak{V}$$

Be micro continuous functions satisfying

$$\mathcal{H}(a, 0) = \mathcal{F}(a) \text{ and } \mathcal{H}(a, 1) = \mathcal{G}(a), \qquad \text{for all } a \in \mathfrak{U},$$

where the functions $\mathcal{F}, \mathcal{G} : \mathfrak{U} \to \mathfrak{V}$ are micro continuous. Then, $\mathcal{H}$ is assigned as a micro homotopy from $\mathcal{F}$ at $\mathcal{G}$, denote by $\mathcal{F} \simeq_{micro} \mathcal{G}$.

If $\mathcal{F}$ is micro homotopic to a constant, then it is called micro null-homotopic. We may think of the function

$$\mathcal{H}: : \mathfrak{U} \times \mathrm{I} \rightarrow \mathfrak{B}$$

as actually being a family of functions $\{\mathcal{F}_t: \mathfrak{U} \rightarrow \mathfrak{B}\}$, for $t \in [0,1]$ where $\mathcal{F}_t$ actually means the function

$$\mathcal{F}_t(a) = \mathcal{H}(\mathrm{a}, \mathrm{t}).$$

Example (3.2): Consider $\mathfrak{U} = \{i, j\}$ whenever $\mathfrak{U}/_{\mathfrak{R}} = \{\{i\}, \{j\}\}$ and $\mathfrak{S} = \{i\}$.Then,

$$\tau_{\mathfrak{R}}(\mathfrak{S}) = \{\mathfrak{U}, \emptyset, \{i\}\}.$$

Let $\mathcal{M} = \{j\}$, and

$$\mathcal{M}_{\mathfrak{R}}(\mathfrak{S}) = \{\mathfrak{U}, \emptyset, \{i\}, \{j\}\}.$$

Assume that $\mathfrak{B} = \{1,2,3\}$ along with $\mathfrak{B}/_{\mathfrak{R}} = \{\{1\}, \{2,3\}\}$ and $\mathfrak{Y} = \{1\}$.So,

$$\tau_{\mathfrak{R}}(\mathfrak{Y}) = \{\mathfrak{B}, \emptyset, \{1\}\}.$$

Let $\mathcal{M}' = \{1,3\}$, then

$$\mathcal{M}'_{\mathfrak{R}}(\mathfrak{Y}) = \{\mathfrak{B}, \emptyset, \{1\}, \{1,2\}, \{1,3\}\}.$$

Consider the functions $\mathcal{F}$ and $\mathcal{G}$ are micro continuous from $\mathfrak{U}$ to $\mathfrak{B}$ defined as

$$\mathcal{F}(i) = 1, \mathcal{F}(j) = 3,$$

and

$$\mathcal{G}(i) = 1 , \mathcal{G}(j) = 2.$$

Let us define $\mathcal{H}: \mathfrak{U} \times \mathrm{I} \rightarrow \mathfrak{B}$ as below:

$$\mathcal{H}(a, 0) = \mathcal{F}(a) \text{ and } \mathcal{H}(a, 1) = \mathcal{G}(a)$$

It is observed that

$$\mathcal{H}(i, 0) = 1, \mathcal{H}(i, 1) = 1$$

$$\mathcal{H}(j, 0) = 3, \mathcal{H}(j, 1) = 2$$

Thus, $\mathcal{H}$ is micro homotopy between the micro-continuous functions ($\mathcal{F}$) and ($\mathcal{G}$).
Proposition (3.3): Let $\mathcal{F}, \mathcal{G}: \mathfrak{U} \rightarrow \mathfrak{B}$ and $\varphi: \mathfrak{B} \rightarrow \mathfrak{D}$ be micro continues functions such that $\mathcal{F}$ is micro homotopic to $\mathcal{G}$. Then $\varphi \circ \mathcal{F}$ is micro homotopic to $\varphi \circ \mathcal{G}$.
Proof. Let $\mathcal{H}: \mathfrak{U} \times \mathrm{I} \rightarrow \mathfrak{B}$ be a micro homotopy between $\mathcal{F}$ and. Then, $\mathcal{H}$ is micro continuous and

$$\mathcal{H}(a, 0) = \mathcal{F}(a) \text{ and } \mathcal{H}(a, 1) = \mathcal{G}(a), \qquad \text{for all } a \in \mathfrak{U}.$$

To prove that $\varphi \circ \mathcal{F}$ is micro homotopic to $\varphi \circ \mathcal{G}$. Let us define $\mathcal{K}: \mathfrak{U} \times I \rightarrow \mathfrak{D}$ such that

$$\mathcal{K}(a, t) = \varphi(\mathcal{H}(a, t)) \qquad \forall a \in \mathfrak{U}, t \in I$$

It is clear that $\mathcal{K}$ is micro continuous since φ is micro continuous. Moreover, we have

$$\mathcal{K}(a, 0) = \varphi(\mathcal{H}(a, 0)) = \varphi(\mathcal{F}(x)) = \varphi \circ \mathcal{F}(a)$$

and

$$\mathcal{K}(a, 1) = \varphi(\mathcal{H}(a, 1)) = \varphi(\mathcal{G}(a)) = \varphi \circ \mathcal{G}(a)$$

Thus, $\varphi \circ \mathcal{F}$ is homotopic to $\varphi \circ \mathcal{G}$.

Proposition (3.4): Assuming maps $\mathcal{F}, \mathcal{F}^*: \mathfrak{U} \rightarrow \mathfrak{V}$ along with $\mathcal{G}, \mathcal{G}^*: \mathfrak{V} \rightarrow \mathfrak{D}$ are micro continuous. Consider composite maps $\mathcal{G} \circ \mathcal{F}$ and $\mathcal{G}^* \circ \mathcal{F}^*$. Then $\mathcal{G} \circ \mathcal{F} \simeq_{micro} \mathcal{G}^* \circ \mathcal{F}^*$ whether $\mathcal{F} \simeq_{micro} \mathcal{F}^*$ as well $\mathcal{G} \simeq_{micro} \mathcal{G}^*$,

Proof. Let $\mathcal{H}: \mathfrak{U} \times \mathrm{I} \rightarrow \mathfrak{V}$ be a micro homotopy between $\mathcal{F}$ and $\mathcal{F}^*$and let $\mathcal{K}: \mathfrak{V} \times I \rightarrow \mathfrak{D}$ be a micro homotopy between $\mathcal{G}$ and $\mathcal{G}^*$. Define a map

$$\mathcal{S}: \mathfrak{U} \times I \rightarrow \mathfrak{D}$$

with

$$\mathcal{S}(a, t) = \mathcal{K}(\mathcal{H}(a, t), t).$$

Clearly, $\mathcal{S}$ is micro continuous since $\mathcal{K}$ is micro continuous. Moreover

$$\mathcal{S}(a, 0) = \mathcal{K}(\mathcal{H}(a, 0), 0) = \mathcal{K}(\mathcal{F}(a), 0) = \mathcal{G}(\mathcal{F}(a))$$

and

$$\mathcal{S}(a, 1) = \mathcal{K}(\mathcal{H}(a, 1), 1) = \mathcal{K}(\mathcal{F}^*(a), 1) = \mathcal{G}^*(\mathcal{F}^*(a)).$$

Thus, $\mathcal{S}$ is a micro homotopy between $\mathcal{G} \circ \mathcal{F}$ and $\mathcal{G}^* \circ \mathcal{F}^*$.

Let $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{S}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{S}))$ and $\left(\mathfrak{V}, \tau_{\mathfrak{R}}(\mathfrak{Y}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{Y})\right)$ be two micro topological spaces, and let $\mathrm{Map}(\mathfrak{U}, \mathfrak{V})$ be the set of all micro continuous maps from $\mathfrak{U}$ to $\mathfrak{V}$. So, the micro homotopy is an equivalence relation on $Map(\mathfrak{U}, \mathfrak{V})$.

Definition (3.5): Suppose the map $\mathcal{F}: \mathfrak{U} \rightarrow \mathfrak{V}$ is a micro continuous. Thereafter $\mathcal{F}$ is said to be micro homotopy equivalence if there exists a micro continuous map $\mathcal{G}: \mathfrak{V} \rightarrow \mathfrak{U}$ such that

$$\mathcal{F} \circ \mathcal{G} \simeq_{micro} id_{\mathfrak{V}} \text{ and } \mathcal{G} \circ \mathcal{F} \simeq_{micro} id_{\mathfrak{U}}$$

Where $id_{\mathfrak{U}}$ and $id_{\mathfrak{V}}$ are the identity maps on $\mathfrak{U}$ and $\mathfrak{V}$, respectively. The map $\mathcal{G}$ is said to be a micro homotopy inverse to $\mathcal{F}$.

It is well-known that the function $\mathcal{F}$ is called micro homeomorphism if its injective, surjective, micro continuous and $\mathcal{F}^{-1}$ is continuous. So, every micro homeomorphism $\mathcal{F}: \mathfrak{U} \rightarrow \mathfrak{V}$ is a micro homotopy equivalence that is simple by taking $\mathcal{G} = \mathcal{F}^{-1}$.

In micro topology, two spaces are considered identical if they are micro homeomorphic. However, this is usually a very difficult to hold. So, we will introduce the notion of micro homotopy equivalent, in order to achieve a somewhat coarser classification.

Definition (3.6): Two micro topological spaces $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{S}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{S}))$ and $\left(\mathfrak{V}, \tau_{\mathfrak{R}}(\mathfrak{Y}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{Y})\right)$ are said to be micro homotopy equivalent or the same micro homotopy type (written $\simeq_{micro} \mathfrak{V}$) if there is a micro homotopy equivalence map $\mathcal{F}$ from $\mathfrak{U}$ to $\mathfrak{V}$.

Proposition (3.7): If $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{S}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{S}))$ and $\left(\mathfrak{V}, \tau_{\mathfrak{R}}(\mathfrak{Y}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{Y})\right)$ are micro homeomorphic spaces then they are micro homotopy equivalent.

Proof. Since $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{S}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{S}))$ is micro homeomorphic to $\left(\mathfrak{V}, \tau_{\mathfrak{R}}(\mathfrak{Y}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{Y})\right)$, there exist micro homeomorphism

$$\mathcal{F}: \mathfrak{U} \rightarrow \mathfrak{V}$$

such that $\mathcal{F}$ is bijective, micro continuous and $\mathcal{F}^{-1}$ micro continuous. We have

$$\mathcal{F}^{-1} \circ \mathcal{F} = id_{\mathfrak{U}} \text{ and } \mathcal{F} \circ \mathcal{F}^{-1} = id_{\mathfrak{V}}.$$

However,

$$id_{\mathfrak{U}} \simeq_{micro} id_{\mathfrak{U}} \text{ and } id_{\mathfrak{V}} \simeq_{micro} id_{\mathfrak{V}}.$$

Thus,

$$\mathcal{F}^{-1} \circ \mathcal{F} \simeq_{micro} id_{\mathfrak{U}} \text{ and } \mathcal{F} \circ \mathcal{F}^{-1} \simeq_{micro} id_{\mathfrak{V}}.$$

Hence, $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{S}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{S}))$ and $\left(\mathfrak{V}, \tau_{\mathfrak{R}}(\mathfrak{Y}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{Y})\right)$ are micro homotopy equivalent.

Proposition (3.8): Micro homotopy type is an equivalence relation on micro topological spaces.

Proof. Let $(\mathfrak{U}, \tau_{\mathfrak{R}}(\mathfrak{S}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{S}))$ and $\left(\mathfrak{V}, \tau_{\mathfrak{R}}(\mathfrak{Y}), \mathcal{M}_{\mathfrak{R}}(\mathfrak{Y})\right)$ and $(\mathfrak{D}, \tau_{\mathfrak{R}}(\mathcal{C}), \mathcal{M}_{\mathfrak{R}}(\mathcal{C}))$ be micro topological spaces. To prove

a) Reflexivity ($\mathfrak{U} \simeq_{micro} \mathfrak{B}$).
The identity map $id_{\mathfrak{U}} : \mathfrak{U} \rightarrow \mathfrak{U}$ is a micro homeomorphism, and thus it is a micro homotopy equivalence.

b) Symmetric ($\mathfrak{U} \simeq_{micro} \mathfrak{B} \Rightarrow \mathfrak{B} \simeq_{micro} \mathfrak{U}$).
Suppose $\mathcal{F}: \mathfrak{U} \rightarrow \mathfrak{B}$ is a micro homotopy equivalence with micro homotopy inverse $\mathcal{G}$. Then, $\mathcal{G} : \mathfrak{B} \rightarrow \mathfrak{U}$ is a micro homotopy equivalence, with micro homotopy inverse $\mathcal{F}$.

c) Transitive ($\mathfrak{U} \simeq_{micro} \mathfrak{B}$ and $\mathfrak{B} \simeq_{micro} \mathfrak{D} \Rightarrow \mathfrak{U} \simeq_{micro} \mathfrak{D}$).
Suppose $\mathcal{F}: \mathfrak{U} \rightarrow \mathfrak{B}$ is a micro homotopy equivalence with micro homotopy inverse $\mathcal{G}$, and $\mathcal{F}^*: \mathfrak{B} \rightarrow \mathfrak{D}$ is a micro homotopy equivalence, with micro homotopy inverse $\mathcal{G}^*$. Using Proposition 3.4 (and the associativity of compositions) the following assertion is readily verified: $\mathcal{F}^* \circ \mathcal{F}: \mathfrak{U} \rightarrow \mathfrak{D}$ is a micro homotopy equivalence, with homotopy inverse $\mathcal{G} \circ \mathcal{G}^*$.

4. CONCLUSION

This study provides a comprehensive definition of micro homotopy and micro homotopy equivalence, yielding significant insights into their properties. These ideas include putting micro-topological spaces into groups based on homotopy and looking into how they relate to classical topology. The results presented here enhance the field of micro topology and underscore its importance in both theoretical and practical mathematics.

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